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# THE ART OF THE CONCEPTUAL

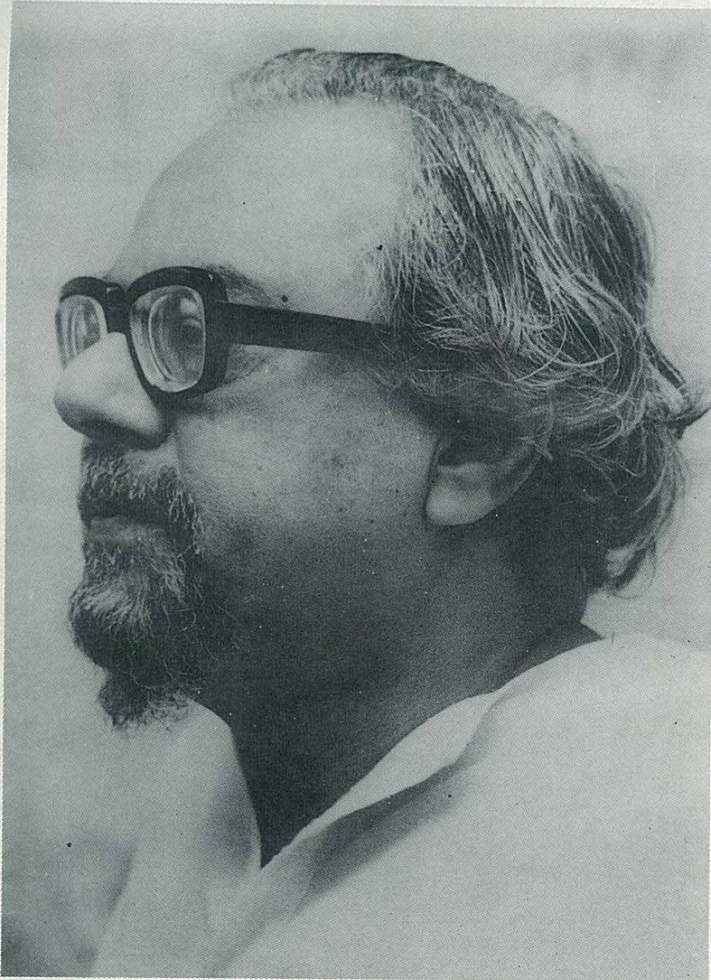
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Explorations in a Conceptual Maze  
Over Three Decades

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DAYA KRISHNA

INDIAN COUNCIL OF PHILOSOPHICAL  
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in association with  
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To

All the Forces  
Transcendent and Immanent  
Personal and Impersonal  
Male and Female  
who  
Have sustained and supported  
The explorations  
All these years  
and  
Who have always given me more  
than  
I could ever give them in return.



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*Philosophy*, February (1965); 'The Active and the Contemplative Values', *Philosophy and Phenomenological Research*, U.S.A. (March, 1969); 'The Self-fulfilling Prophecy and the Nature of Society', *American Sociological Review*, U.S.A. (1971); 'It Can't Be Said So What?', *Indian Review of Philosophy*, Vol. 1, No. 4 (1972); 'Appearance and Reality', K. S. Murty and K. R. Rao (eds.), *Current Trends in Indian Philosophy*, Asia Publishing House (1972); 'The Concept of Revolution—An Analysis', *Philosophy—East and West* (July, 1973); 'Logic and Ontology', Margaret Chatterjee (ed.), *Contemporary Indian Philosophy, Series Two*, Muirhead Library of Philosophy, George Allen and Unwin. (1974); 'Arts and the Cognitive Enterprise of Man', *The Visva-Bharati Quarterly*, Vol. 41, Nos. 1-4 (1975-76); 'Culture as a Field of Interdisciplinary Study', *International Social Science Journal*, 1977; 'Anthropology—The Bonded Science?' *New Quest*, May-June, 1983; 'Self and Its Representations in Literature' *The Literary Criterion* (Vol. 20, No. 3, 1985).

## PREFACE

To be asked to select one's articles for publication by the Indian Council of Philosophical Research is, in one sense, to be reminded that perhaps it is time for stock-taking, to be accountable to the world of fellow-philosophers and the academic community at large and display for their critical attention and evaluation what one had been doing all these years in an *ad hoc*, piece-meal manner. One does write and publish an occasional piece which might be noticed or not by one's colleagues, but to be asked to collect all, or most of them, at one place is to be asked to give an account of what one has intellectually done all these years. And, even if no one else cares to look into the collection, one cannot escape it oneself as the very process of choosing, editing, arranging, correcting, proofreading makes one ask: what does it all add up to, and was it really worthwhile?

The earliest article included in this collection, 'An Attempted Analysis of the Concept of Freedom' was published in June, 1952 and the latest 'Self and Its Representations in Literature: Some Epistemological Problems' in 1985—a period spanning more than thirty years, bridging the dawning horizons of youth to the setting horizons of age, a long time in the life-time of an individual, scarcely noticeable in the history of thought.

Looking back, it seems that there is a perennial concern with certain central philosophical issues in these articles—the relation of logic to reality and its relevance to philosophy or philosophizing, the multifariousness of values and their essential conflict with one another, the essential irreducibility of diverse realms and the concepts and categories through which we demarcate them, the centrality of consciousness and the strange fact that beliefs tend to bring corresponding realities into being through the actions they influence, the strange and paradoxical nature of social reality as the continuing creation of a plurality of free beings, each simultaneously a subject and an object, an agent and a recipient, all rolled into one.

These themes have been explored elsewhere in my writings also.



Sometimes, the reference has been given in the articles themselves; sometimes not. Many of the issues relating to the understanding of social reality, for example, have been extensively discussed in *Social Philosophy—Past and Future*,<sup>1</sup> *Considerations Towards a Theory of Social Change*,<sup>2</sup> *Political Development—A Critical Perspective*,<sup>3</sup> and the *Development Debate*.<sup>4</sup> The issues relating to logic and reality and the relevance of logic to philosophy have not only been discussed focally in the first seven articles included in this volume, but also in *The Nature of Philosophy*,<sup>5</sup> and in a book edited by me entitled *Modern Logic: Its Relevance to Philosophy*.<sup>6</sup> These issues relate to what has come to be called *Philosophical Logic* now-a-days and they still seem to me to formulate the issues in a focal manner.

The articles selected for publication in the volume have been classified under three major headings: *Logic and Epistemology*, *Moral Philosophy* and *Social, Political and Economic Philosophy*. But these are only broad headings to indicate the primary area in which the issues discussed in the articles may be said to fall into. There are bound to be overlappings and some of the articles could easily have been placed under a different heading. Similarly, the articles in each part have been arranged in a logical and not in a chronological order. This has resulted in some anomalies which the reader is bound to notice if he reads the articles in a sequential manner. In some cases, it did not seem quite clear as to what should be the logical order or where exactly an article should be placed in the series or even under which heading. The articles on Sorokin and Northrop, for example, could as well be included under 'Social Philosophy' as under 'Logic and Epistemology.' In any case,

1. Daya Krishna, *Social Philosophy—Past and Future*, Simla, Indian Institute of Advanced Study, 1969.
2. Daya Krishna, *Considerations Towards a Theory of Social Change*, Bombay, Manaktalas, 1965.
3. Daya Krishna, *Political Development—A Critical Perspective*, Delhi, Oxford University Press, 1979.
4. Fred W. Riggs and Daya Krishna, *Development Debate*, Jaipur, Printwell Publishers, 1987.
5. Daya Krishna, *The Nature of Philosophy*, Calcutta, Prachi Prakashan, 1955.
6. Daya Krishna, D. C. Mathur and A. P. Rao (eds.) *Modern Logic: Its Relevance to Philosophy*, Delhi, Impex India, 1969.

the articles are listed chronologically in the 'Acknowledgements' so that the reader may judge of the development of thought on an issue over a period of time, if any. It may, however, be borne in mind that the chronological sequence gives only the sequence in which the articles were published in the various journals and not the one in which they were actually written. Different journals take different time in publishing an article after acceptance and with some journals the process of acceptance itself may take time depending upon the procedure adopted. Besides this, there may occur unpredictable reasons for delay in publishing for which no one is exactly responsible. The article entitled 'Self-fulfilling Prophecy and the Nature of Society', for example, had to wait several years for publication as the editor of the *American Sociological Review* thought it would be best to publish it with Prof. Merton's reply as it had raised certain basic issues regarding what he had written on the subject. And, as Prof. Merton had agreed to reply to the points made in the article, he decided to defer its publication till such time as his *Reply* was received. But, for some reason or other, Prof. Merton kept on postponing writing his *Reply* for years till one day I received a letter from the editor that as he was leaving the editorship of the Journal, he had decided not to wait any longer and was publishing the article in the next issue of the Journal, the last under his editorship.

The story of Western thinkers' response to a basic criticism of their work is interesting as it reveals a strange sort of resistance to come to terms with a foundational critique of their work, particularly from persons belonging to other cultures. Prof. Northrop at Yale did not seem to have known of the critique published on his theory of concepts in *The Philosophical Review*, published from Cornell. Prof. Lazerowitz has gone on writing as if nothing had been written regarding his whole methodology of dealing with philosophers and philosophical problems in the pages of the *Mind*, though personally he has complained to friends and wondered why I have been so 'hostile' to him. And this, in spite of my writing at the end of the article, 'I should confess that I myself am not highly impressed by such sort of refutations, but if I have taken the trouble to play the game with Lazerowitz's work, it is only that he may come to share my dissatisfaction with the type of arguments he so often employs against other thinkers' work.'



The story is not confined to Merton, Northrop and Lazerowitz alone. The critical examination of the whole notion of 'political development' and of each of the criteria developed by Almand in a full-length work entitled *Political Development—A Critical Perspective* has met the same fate. So also an earlier paper 'Concept of Political Development in American Political Science in the Sixties' published in Joseph Fischer (ed.), *Foreign Values and South East Asian Scholarship*, University of California Press, 1973. But there have been notable exceptions also. Prof. Sorokin not only took serious note of the article whose offprint I had sent him and invited me for dinner to discuss it, but took the trouble of replying to it at length in the book edited by Philip J. Allen<sup>7</sup> even though there was no necessity of doing so as the article had been published elsewhere and was not a part of the book in which his reply was to be published. Prof. Fred W. Riggs' replies to my comments resulted in a debate lasting for years in the pages of *Administrative Change*, which were later collected and published under the title *Development Debate*.

Some other articles have had a different history about them. The article on 'Surplus Value, Profit and Exploitation' evoked considerable interest after its publication in the *Review of Economic Studies*, and was reprinted later in many other journals, including the multi-lingual quarterly *Diogenes* published under the auspices of the International Council of Philosophy and Humanistic Studies, with the support of UNESCO. Its publication in a UNESCO-supported journal evoked protest from the USSR, a far cry from the *Glosnost* situation of these days under Gorbachev in that country. Yet, in spite of all this, I do not know of any serious attempt at meeting the arguments offered there by Marxist thinkers in India or abroad. On the other hand, the review article 'Anthropology—A Bonded Science?' seems to have struck a sympathetic chord amongst some Indian social scientists. But it seems also to have created an impression on Western students of India that I am not very favourably inclined, if not exactly hostile, to their enterprise about India. Positively or negatively, the article has had some impact—and that is one reason why I have included it in this collection. The other, and more important, reason, of course, is that I think the points made in that paper are of sufficient

7. Philip J. Allen (ed.) *Pitirim A. Sorokin in Review*, Durham, 1963.

significance to be known to the wider audience of social scientists in India. Until and unless the West becomes an *object* of study of non-Western social scientists to the same extent, and in the same manner, as the non-Western world has been studied by the Western students of these societies and cultures, not only no balance will be achieved in the comparative study of societies and cultures, but the puerility and perversity of much of what is being done shall not be exposed.

The West, of course, is not prepared to welcome such a reciprocal enterprise to redress the balance, or even to admit its feasibility or desirability. I do not think it has given up the position taken by Claude Lévi-Strauss in his famous lecture at the Smithsonian published in the April, 1966 issue of *Current Anthropology* when he rejected the suggestion that 'the west allow itself to be "ethnographized" by those for whom we were once solely the ethnographers'. The flavour of the full sentence in which the suggestion has been articulated has to be read to be believed. It reads: 'The suggestion has been made that in order to render anthropology *less distasteful to its subjects* it will suffice to reverse the roles and *occasionally allow* ourselves to be "ethnographized" for whom we were once solely the ethnographers.'<sup>8</sup> Imagine the arrogance and disdain involved in the very articulation of the suggestion. No, Prof. Claude Lévi-Strauss does not want such a situation to happen, even 'occasionally'. And, what is his reason for the refusal. In his own words, 'Well-meant as it undoubtedly is, this solution appears to me naive and unworkable, as though the problems were as simple and superficial as those of children unaccustomed to playing together, whose quarrels can be settled by making them follow the elementary rule: "Let me play with your dolls and I shall let you play with mine"'. To arrive at an understanding between people who are not merely estranged from one another by their physical appearances and their peculiar ways of life, *but also stand on an unequal footing to one another*, is a different question altogether.'<sup>9</sup> So anthropological studies can never be done by those who stand in an inferior relationship to those who are superior to them in power-relations. The converse, of course, is not

8. Claude Lévi-Strauss, 'Anthropology: Its Achievements and Future', *Current Anthropology*, Vol. 7, No. 2, April 1966, p. 125; italics mine.

9. *Ibid.*, p. 126; italics mine.



being denied. Rather, it is being asserted that it is only those who are superior in power may study those who are inferior to them. The learned Professor is explicit on this point. He does not mince his words. He writes, 'Anthropology is daughter to this era of violence: its capacity to assess *more objectively* the facts pertaining to the human condition reflects, on the epistemological level, a state of affairs in which (*one*) part of mankind treated the other as an object.'<sup>10</sup> He is also candid enough to admit that 'It is not because of its mental endowments that only the Western world has given birth to anthropology, but rather because exotic cultures, treated by us as mere things, could be studied, accordingly, as things.'<sup>11</sup> And, how could this situation come about that members of one culture could study members of another culture as mere objects? According to Claude Lévi-Strauss, it was 'the outcome of a historical process which has made the larger part of mankind subservient to the other, and during which millions of innocent human beings have had their resources plundered and their institutions and beliefs destroyed, whilst they themselves were ruthlessly killed, thrown into bondage, and contaminated by diseases they were unable to resist.'<sup>12</sup> As the non-Western world has not yet been able to do this to the West, its anthropologists are not, in principle, in a position to study Western societies anthropologically, that is, as mere objects.

I had not read Lévi-Strauss' address when I had written the *Review-Article* included in this volume. But I had felt there was something wrong in the situation and as I had not been sufficiently socialized into the intellectual culture of anthropology, I reacted in the way a normal non-Western intellectual would, or should. Today, after reading Lévi-Strauss' address, I am surprised at the utter lack of any response to it amongst the intellectuals of the non-Western world in general and the anthropologists in particular. Perhaps the intellectual acculturation has penetrated so deep into the psyche that the unbelievably perverse partisan positions of many of the most outstanding intellectuals of the West goes unnoticed and unchallenged. In fact, the international conference

10. Ibid., p. 126; italics mine.

11. Ibid., p. 126; italics mine.

12. Ibid., p. 126; italics mine.

called by the Smithsonian and the Wenner-Gren Foundation for Anthropological Research to discuss the issue raised in Lévi-Strauss' address did not even discuss this part of his contention. William C. Sturtevant who prepared the conference report and which was published in *Current Anthropology* mentions the fact of 'non-discussion' of the issue by the Conference and draws the strangest conclusion from it—that the Conference rejected it. In his own words, 'The Conference *implicitly* rejected—by failing to consider his reasons—Lévi-Strauss' *warning* that reciprocal study of Western societies by non-Western anthropologists is not the answer.'<sup>13</sup> What sort of reasoning is this which glosses over the stunning fact that anthropologists drawn from all parts of the world, including such non-Western countries as Taiwan, Bogota, Zambia, Japan, India,<sup>14</sup> Korea and Uganda, did not even react to Lévi-Strauss' contention, let alone discuss it. Such a colossal failure *demand*s a socio-cultural study of its own, but as perhaps the phenomenon is so widespread, nobody noticed it. But, note the watering down of Lévi-Strauss' contention. What was being propounded, almost as a matter of principle, is treated only as a *warning*, to be heeded or not.

A preface is, perhaps, not the place to go into such issues. But the background and the necessity behind the inclusion of a *Review-Article* in a collection of articles mainly devoted to conceptual exploration of philosophical issues had to be explained to some extent.

The articles cover a long period and deal with diverse issues. They were published in various Journals from different countries and hence were not easily accessible as they are seldom available in most university libraries in India. I myself had difficulties in locating the issues and getting photo-copies made of some articles whose copies I did not have with me, for some reason or other. In fact, had it not been for the gentle but persistent insistence of Prof. D. P. Chattopadhyaya, the Chairman of the Indian Council of Philosophical Research, and his frequent reminders to expedite the whole thing, it is difficult to believe that the task of collecting

13. *Current Anthropology*, Vol. 8, No. 4, Oct. 1967, p. 356; italics mine.

14. India was represented by Surajit Sinha from Calcutta and Iravati Karve from Pune.



them in a publishable form would ever have been completed. On him, therefore, lies all the responsibility and he alone deserves all the praise or the blame for rendering the ephemeral and the inaccessible into the relatively more permanent and accessible and, thus, generating the illusion in the author that what was not much noticed or attended to would now be paid the critical attention, it perhaps deserves. To the editors and publishers of the various journals thanks are due for giving the formal permission to include the article/s in this collection. And, to Shri Buddhadev Bhattacharya, who has taken such pains and given such personal attention to the printing and publication of the whole thing, I can only say 'Let all authors find such friends who have the skill, the dedication and the judgment in the field of printing and production as Buddhadev has'.

And what about the stock-taking and the worthwhileness? Well, I find the issues still important and the arguments reasonably convincing to place them before the reader once more for his critical judgment in the matter.

*Jaipur*

DAYA KRISHNA

## I. Logic and Epistemology



## LOGIC AND ONTOLOGY

Logic is supposed to be neutral to ontology. The search for something which will hold true in all possible worlds, or even if there were to be no world at all, has had a perennial fascination for the philosophical mind. The criteria of universality and necessity which seemed to belong to mathematics made it the ideal paradigm for knowledge, at least in the western tradition of thought deriving from the Greeks, particularly Pythagoras and Plato. But however universal and necessary mathematical knowledge may seem, it certainly cannot lay claim to the exhaustiveness of such knowledge. Not only does mathematical knowledge continue to grow, but no one seems to think that it can ever stop growing. Though it may grow as a matter of fact, there seems nothing in principle which will ensure that it must be so, specially with the view most philosophers of mathematics entertain about mathematical knowledge.

The claim of mathematics to be considered as knowledge has always posed problems for so-called tough-minded philosophers with the empiricist conviction that ontological status be granted only to that which is encountered in sense experience. They find it difficult to deny it the status of knowledge and even more difficult to accept that it is such. To accept it as knowledge is inevitably to raise the question, 'knowledge of what'? It is difficult to accept that it is knowledge of that which is encountered in sense experience, as this would not only rob it of that element of universality and necessity which has been the reason for its perennial fascination for the rationalist philosophers in the west, but also make the whole field of transfinite mathematics impossible in principle, unless, of course, it were to be accepted that infinity can be directly encountered in sense experience. If we add the perspective of growth to our reflection on the kind of knowledge that mathematics happens to be, then it would be even more difficult to accept that it is not knowledge of anything whatsoever.



The possibility of growth in principle with respect to the sort of knowledge that mathematics happens to be, appears *prima facie* to be in conflict with the purely logical and deductive nature which is usually ascribed to it. In fact, a certain sort of finality seemed to characterize Euclidean geometry and Aristotelian logic from the earliest point of their inception until almost the middle of the nineteenth century when the elaboration of non-Euclidean geometries revealed the purely postulational character of the Euclidean axioms which were supposed to have been self-evidently true. The assimilation of geometry to algebra earlier in Descartes and the subsequent attempts to translate each branch of mathematics in terms of the others revealed the basic unity of all mathematics and the articulation of this unity in terms of the notion of sets and the possible operations that could be performed on them. The deductive character of various geometries revealed the foundational nature of the process of deduction and thus made many people seek in logic the foundations of mathematics. The same thing was achieved by the formal properties of the operations on sets which were shared by certain other operations which happened to be more obviously logical in character.

Logic has generally been regarded as being concerned with relations between sentences or propositions. Mathematics, on the other hand, may be said to be concerned with relations between numbers. The ontological question, then, in the case of the two may be said to relate to the status we wish to accord to sentences or propositions on the one hand and to numbers, on the other. To the extent that propositions are regarded as pseudo-referents for sentences, the same may be said with respect to numbers as the referents of marks which are supposed to stand for them. The problem, then, shifts to the so-called semantic function and the issue relates to the question whether all referents have to be of the same ontological type. The fact that the sign or symbol which performs the referential function happens to be sensory in character is irrelevant to the issue. It has perhaps necessarily to be such for the purpose of communication, unless one is prepared to accept the possibility of direct telepathic communication amongst human beings. What matters, however, is that which is referred to and whether it is only of the type that is apprehensible by the senses.

The whole thrust of recent philosophical thought in Anglo-

American countries seems to be towards the devising of ways and means to escape the cognitive necessity of postulating the reality of entities other than those which are, or could be, directly apprehended in or through sense experience. Quine and Goodman's almost half-conscious confession epitomises that.<sup>1</sup> Yet the naive equation of all that is apprehended through the senses with reality flies in face of the fact that the illusory appears as real. As Shalya has forcefully argued recently, the concept of reality itself is a theoretical concept and is correlative to a systematic context.<sup>2</sup> Otherwise, the distinction between the real and the unreal will collapse, and there would be no such thing as unreality at all. Yet, if what is to be considered as 'real' were to be the result of the acceptance of a criterion, then we would have the problem of the justification of the criterion on our hands unless we are prepared to grant the possibility of multiplicity of criteria with no rational grounds for choosing between them.<sup>3</sup>

The fear of granting ontological reality to abstractions that seems to haunt so many Anglo-American philosophers in this ultra-abstract age may find some explanation in the acute observation of K.C. Bhattacharyya that to the extent to which we identify ourselves with the body, we cannot but help regarding the sensory world alone as pre-eminently real and all the rest as its shadow. On the other hand, the very moment we begin to identify ourselves with mind or intellect or reason, the perceptual world begins to appear as a shadow of the world grasped by faculties other than that of sense experience.<sup>4</sup> As in the normal human knowing process, senses and intellect both play an inalienable role, and as man is simultaneously both body and mind, what is regarded as real and what as the shadow depends on the valuational primacy one accords to the one or the other. The ontological denigration of abstraction, then, may be seen as a correlate of the sensate value system of modern man which regards body as the primary reality of man. However, just as for the rationalist-idealist philosophers,

1. 'Any system that countenances abstract entities we deem unsatisfactory as a final philosophy.' (*Journal of Symbolic Logic*, December 1947).
2. Yashdev Shalya, *Jñāna aur Sat* (Delhi: Rajkamal Prakashan, 1967).
3. See the discussion in my book *The Nature of Philosophy* (Calcutta: Progressive Publishers, 1955), Chapter III.
4. K.C. Bhattacharyya, *Studies in Philosophy*, Vol. II, (Calcutta: Progressive Publishers, 1958).



body with its correlate the sensory world is the focal point of ontological attack, so is the empiricist philosophers' battle against abstractions, which, like some of the ancient demons of Hindu mythology, rise and multiply faster than they are killed.

The attempted reduction of mathematics to logic reduces the ontological issue to the referent of logic alone, and as logic does not ostensibly seem to be about any specific entities, the necessity of granting ontological status to anything other than those encountered in sense experience evaporates also. Logic seems to be concerned with the elaboration of rules which, if followed, would always ensure the deduction of true consequences from the premises. The rules are, to use the usual terminology, "truth preserving." How do we know whether the rules are such as to necessarily ensure truth preservation? The only answer that could possibly be given might relate to the fact that no one has yet been able to derive a false conclusion from true premises by following the rules, or that the term 'true' means derivability according to the rules, thus making it impossible in principle to call any conclusion "false," if it were derived according to the rules. In the first case, the argument would be essentially inductive in nature and would suffer all the problems and the limitations which inductive reasoning usually does. It could not preclude the possibility that someone could show such a derivation in the future and thus show the rules to be incapable of achieving what they were supposed to achieve. If, on the other hand, we adopt the second alternative to obviate this contingency, the notion of truth becomes purely syntactical and the characterization of logic as truth preserving loses all sense. For, if truth means derivability according to rules, then neither premises nor conclusions could be regarded as true or false on their own.

The desire to provide a semantic underpinning to logic derives perhaps from the desire to make logic applicable and thus useful. The parallel analogue with mathematics would, however, also raise the same questions with reference to logic. There would have to be a distinction between pure and applied logic and Einstein's famous remark about the relationship between pure and applied mathematics would also apply to their relationship.<sup>5</sup> On the other

5. 'To the extent it can be applied, it is not exact and to the extent it is exact, it cannot be applied'.

hand, the practically oriented may take comfort from the fact that whatever the theoretical difficulties, the relationship between pure and applied logic will continue to be established and may be as fruitful as in the case of mathematics. However, such a consideration, if taken seriously, might strike at the root of most philosophical perplexities which, in any case, happen to be purely theoretical in nature. The application question, on the other hand, if closely analysed, may reveal some special problems in the case of logic which do not arise in the case of mathematics. The usual field of the application of logic is in the realm of the arguments which people give. If we admit that the substitution of concrete premises in place of the usual sentential variables makes the slightest difference to the correctness of the formalized argument, then the so-called application will become completely nugatory in practice. To put it in another way, there can be no such thing as the "approximation" of a concrete argument to a purely formalized one, as is usually the situation in the case of the application of mathematics. Concrete points and lines can be said to approximate the idealized points and lines of geometry and it is this feature that leads to "degrees of precision" in scientific measurement which can be carried to any length depending upon our needs.<sup>6</sup> Concrete arguments are either correct or incorrect; there is no middle ground for them, no more or less, no indefinite approximation to an ideal which is never reached.<sup>7</sup>

The same appears to be the case with respect to what may be called the new field of "applied logic" as contrasted with what may be considered as the traditional one. Hardly anyone would think these days of applied logic as consisting of arguments where concrete premises are substituted for sentential variables. Rather, it

6. There are supposed to be some intrinsic limits to measurement deriving from Heisenberg's principle of uncertainty and perhaps the absoluteness of the velocity of light as postulated in Einstein's theory of relativity, but that does not basically affect our contention.
7. The problems arising from the substitution of concrete sentences for the sentential variables has been discussed in the author's article 'Law of Contradiction and Empirical Reality', *Mind*, April 1957, and "Types of Coherence" *The Philosophical Quarterly*, July 1960. For an extensive critique of the latter article, see Dharmendra Kumar, "Types of Coherence—a Refutation," *The Visva-Bharati Journal of Philosophy*, Vol. IV, 1965.



is in computer programming where it is the logical connectives that are given a mechanical or electrical analogue, that we find the field of applied logic *par excellence*. However, the computer, by being built around the function of the logical connectives, only simulates the formalized argument schema and behaves fundamentally like the human manipulator in a formalized derivation according to the transformation rules of a deductive system. It, thus, is more akin to the usual mathematical calculator and should, in the strict sense, not be considered as belonging to the field of applied logic at all.

The dependence of the notion of truth preservation on the prior notion of truth, and that too in its semantic aspect, raises not only a host of issues which have not been attended to in the literature on the subject, but also makes logic depend upon ontology in a sense which most pure logicians may not consider very welcome. Traditionally, one could not think of a valid argument, unless it conformed to the syllogistic rules, which required, amongst other conditions, the presence of at least one universal premise for its validity. As is well known, however, it is impossible in principle to establish the truth of a universal proposition unless it is just a summation of a finite set of instances, however large they may be. If it is impossible to establish the truth of a universal proposition and it is also essential that it function as one of the premises in a valid argument form, then it is obvious that it could never be known whether the rules had ensured truth preservation or not. The usual way out is to assume the truth of the premises, ignore the fact that the truth of at least one of them cannot be established in principle, assert an 'if-then' relationship between the premises and the conclusion, and establish its validity on the basis of its being in accordance with the rules of derivation. Yet, however convenient this solution may appear in practice, it is obvious that it cannot justify the validity of the rules themselves on the ground that, if followed, they will always ensure the impossibility of deriving a false conclusion from true premises. Thus, in a fundamental sense, logic in its traditional presentation could not ensure the fulfilment of its central task which was supposed to be that of truth preservation.

Does modern logic fare any better in the performance of that very function which it agrees is the central task of all logic, whe-

ther traditional or modern? In a sense, the concept of 'truth-functionality' may be regarded as the central point of modern logic, and the explication of the concept of derivation or proof its central concern. Truth-functionality is itself a function of the notion of 'truth values' on the one hand and that of 'truth operator' or 'logical operator' on the other. Truth values, as everyone knows, may be as many as one wants as long as their number is more than one—logic would perhaps have no function if sentences could not have any other truth value except one.<sup>8</sup> As for logical operators, the one logically significant thing about them is the number of minimum sentences they require to form a new sentence. As this can vary from the minimum of one sentence to as many as one chooses to imagine, we have the whole range from monadic to n-adic logical operators. The actual number of logical operators, then, is a function of the decision with respect to the number of truth values we accept for our logical system and the minimum number of sentences which we want our operators to operate upon. However, as any choice is bound to be arbitrary in the matter, and as it is basically a selection out of the infinite possibilities between which there can be no logical grounds of choice, any particular choice is bound to be a special case of generalized logic.

The situation arising from this indefinite infinity of logical operators is usually met in the literature by showing the definability of *all* such operators in terms of a few fundamental ones, such as conjunction and negation or disjunction and negation. This definability or rather describability in terms of a few operators does not annul the distinction between different operators and thus the proposed solution can only be regarded as spurious in character. The operators continue to be different, whether they are described in one way or another.

The indefinite multiplicity of logical operators, each distinctive from all the others and irreducible to any one of them in any significant sense of the term, arises due to different factors. One relates to the minimum number of sentences which a particular operator requires to form a new sentence, while the other relates to the pos-

8. This is not entirely correct, as logic without the operator of negation has been tried. The attempt, however, only shows that very little can be done with it.



sible truth values that a sentence may have. As there is no intrinsic limit to either of these, the indefinite multiplicity of logical operators would follow from either of them, taken either singly or jointly. The question regarding the meaningfulness of such an indefinite extension on either count should strictly be treated as an extra-logical question on the truth-functional view of logic. Yet, such is the hangover of these extra-logical interests even for the so-called truth-functionalists that they continue to ask questions such as what is the meaning of truth values which a sentence is supposed to have in many-valued logics. The usual reassurance given is in terms of finding the interpretation in terms of probability values which may range anywhere between 0 and 1. The limits are supposed to provide the usual truth values of falsity and truth in a two-valued logic, but it is obvious that there is no necessity of such an interpretation. The indefinite multiplicity of possible truth values could as well be correlated with natural numbers ranging from one onwards, without making the situation any different except that there will be no upper limit to the series and if the notion of truth were to be identified with such a limit, there would be no such thing as truth.

The objection that the number of numbers in the two series is the same, as they can be put in a one-one correlation, is besides the point. In fact, it is wrong to think that because the two series are the same in respect to the number of their numbers, they are the same in other respects also. The point made above that while the one series may be said to have an upper limit, the other cannot, suggests a basic difference in at least this aspect of the matter. But this is not the only difference. We may, for example, significantly ask what is the next number in the series of natural numbers, although it would be meaningless to ask the same question regarding the series of rational numbers. Now, if we accept Ryle's criterion that if the same question cannot be meaningfully asked of two sets of entities then they should be held to be basically different, we would have to accept that the series of natural numbers and the series of rational numbers are fundamentally different in spite of the fact that they have the same number of numbers. The situation on Ryle's criterion is, in fact, even more desperate. The criterion demarcates types or categories of objects. If so, the two series would have to be assigned to different categories on Ryle's

criterion. The philosophers of mathematics may take their choice and reject Ryle's criterion or accept the discomfoting conclusion that the two series are categorically different.

The redundancy of the probability interpretation of n-valued logic is, however, only one aspect of the matter. The deeper problems lie in respect to the notion of truth-functionality itself when considered in relation to n-valued logic. In two-valued logic, whether the logical operator happens to be monadic, dyadic or n-adic, there is always an operator which in all conditions gives T and another which in all conditions gives F. These are known as tautology and contradiction, respectively. On the same analogy, in an n-valued logic we would have n-types of operators which in all cases would give the same value because of the nature of the operators themselves. Yet, what exactly would it mean for operators other than those that are tautological or contradictory to have the same value in every case? To ask this question is to realize that either the notion of truth-functionality has little sense as far as logic is concerned or that the concepts of truth and falsity are irrelevant for the type of study that modern logic aspires to be. The usual two-valued truth-functional logic with its values of truth and falsity symbolized as T and F masks this dilemma which comes into the open the moment logic is generalized as a truth function of n-values. It is then that the arbitrariness of the names designating the so-called truth values is revealed, for the moment we have more than two of such values, we realize that as far as truth-functional logic is concerned, what is important is their number, that is, whether they are one or two or three or four, or any other number. Any further characterization is basically irrelevant.

If the notions of truth and falsity are irrelevant to truth-functional logic, then what could possibly be meant by truth preservation being the central task which the rules of logic are supposed to ensure? There is no truth to preserve as the so-called truth values are essentially a misnomer. It is, if we may be allowed to use a slightly modified form of Ryle's well-known phrase, a systematically misleading name, but once this is realized, some of the restrictions imposed on the sentential variables on which the truth-functional operators are supposed to operate, would also seem otiose. For example, there would seem no reason for saying that the sentential variables should range only over declarative



sentences, as they alone can be true or false. In a certain sense, the extension of logic to such fields as the logic of imperatives suggests that the restriction is basically unjustified. The reason why it has usually been regarded as a necessary restriction may then be seen to follow from the mistaken understanding about the nature of two-valued logic. If there can be such a thing as a logic of imperatives and if imperative sentences are not supposed to be either true or false, then the whole idea of logic as the elaboration of rules which are truth preserving falls to the ground. There is, in such a case, not even the pseudo-reference to truth which at least seemed a character of the sentence types with which two-valued truth-functional logic was supposed to deal. An imperative sentence is not true or false, and hence the question of whether what follows from it is true or false is besides the point.

The extension of logic into other fields does not merely bring into question the notion of truth preservation as the central task of logic in the manner it has been traditionally conceived. In some of the developments, it may be said to question the very centrality of the notion of truth functionality for logic, which was supposed to have been given to it by modern developments in logic. If, for example, there is such a thing as a logic of belief statements such as Hintikka and others have tried to develop, and if, 'I believe that' is not a truth-functional operator as has been usually contended, then it is obvious that logic need not concern itself with truth-functional relations at all.

The meeting point of what may be called the ontological and epistemological issues with respect to logic may perhaps be found still better in the case of what is called 'quantificational logic'. This logic relates to those types of argument-forms where the derivation of the conclusion depends upon the quantificational character of the premises concerned. Here all the restrictions on the dropping and adding of quantifiers are meant to ensure that no false conclusion is derivable from premises that happen to be true. How could it ever be proved that the rules do ensure this? As already pointed out in the case of the supposedly necessary universal premise in the traditional syllogistic argument, what is required necessarily happens also to be in principle impossible of being ever established as true. The difficulties arising from this situation are

not confined to traditional logic alone. Rather, the troubles besetting modern logic seem to be compounded still more.

First, except for a few eccentrics like Fisk, almost all the modern logicians seem to argue for a fundamental ontological difference between the two quantifiers. The universal quantifier, according to the new orthodoxy, should be interpreted as ontologically neutral. In other words, its truth should not be taken to entail the existence of any entities whatsoever. The converse is supposed to be the case with the so-called existential quantifier. Yet the two are supposed to be interchangeable. In fact, the rules for the translation of the one into the other are usually given along with the contention that the two are also radically different in their ontological commitments. Perhaps it is felt that there are no difficulties because the translation is done in terms of a negation of the other quantifier along with a negation of the property or properties which were supposed to be possessed or not possessed, as the case happened to be. Also, when translated into the usual language of meaning, the two appear to be equivalent, even though according to many of these logicians themselves it is difficult to have any satisfactory criteria of synonymy. But no such considerations or appeal to meaning seem relevant to the issue concerned. The ontological difference was intended to relate to the form alone, but if the forms themselves are considered to be interchangeable, then there seems no ground for contending that there is an essential ontological difference between the two.<sup>9</sup>

The problems arising out of the use of the quantifier 'all' are, however, not confined only to those mentioned in the last paragraph. They arise even more insistently and openly in what is known as the logic of classes or the set theory. In quantificational logic they are usually masked by the notion of a 'universe of dis-

9. An analogous problem, though without ontological overtones, arises in relation to traditional logic when it formulates a rule that from two negative premises no conclusion can be drawn, forgetting that by its own rules of obversion, any proposition could be changed into another equivalent proposition whose quality is opposed to that of the former. Thus every negative proposition can be transformed by obversion into an affirmative proposition and vice versa. Thus either no conclusion could ever be derived from any premise or it could always be so derived, for all affirmative propositions could be turned into their equivalent negative forms and all negative propositions into the equivalent affirmative ones.



course', which is just a euphemism for saying that we are not talking about 'all', but only about 'all of a certain type'. However indefinite may be the extension generated in this way, it is defined by the possession of a property or properties, as the case may be. But the notion of an unrestricted 'all', not confined to the possession of this or that property, comes into the open only when we begin to talk of classes and discover the universal class.

What exactly is this universal class? It is usually considered to be a result of the operation of union on a class and its complementary class. As the complementary class is formed of all those members which do *not* possess the said property, the class formed of all those who either possess or do not possess a property is supposed to be the universal class of which everybody has to be a member since either one possesses the property or one does not possess it. As the non-possession of the property is defined by exclusion in the manner of the Buddhist doctrine of *Apoha* it may be taken as exhausting the universe in the manner of the traditional formulation of the Law of Excluded Middle as 'A must either be B or not-B'. But it is well known that the law is not accepted by everybody and to the extent it is rejected, it would affect the notion of a universal class which, at least in its usual formulation, seems built upon it.

The 'universality' of the universal class, however, does not depend only on the acceptance or rejection of the Law of Excluded Middle. It may be affected significantly by other considerations also. For example, is the notion of 'exclusion' to be applied only to those types of objects to which the property may significantly be said to apply? In other words, is it to be confined only to what is called the 'universe of discourse' in the literature on the subject? For example, does the complementary class of all things which are unified by the non-possession of the property 'blue' include such things as music or-2? In case it does not, the universal class is universal only in name, as everything is not its member. On the other hand, if it is allowed to have them as its members, not only does the whole notion of 'pseudo-question' along with that of meaninglessness on which so much of modern philosophizing rests, fall to the ground, but some serious difficulties are created for the traditional formulation of the Law of Contradiction also. Usually, it is formulated as 'A cannot be both B and not-B', but

if not-B is interpreted without the restriction imposed by the universe of discourse, it would lead to the absurd situation that something cannot be, say, both blue and square as 'square' would have to be included under the domain of 'not-blue'.<sup>10</sup> If, on the other hand, it is formulated in its modern form as 'A declarative sentence cannot be both true and false', then its formal truth would be only a consequence of the prior assumption that truth and falsity are the sort of properties that belong to sentences and that they are exclusive of each other. Yet, whether any properties are exclusive of each other is a factual question, and no *a priori* decision can be reached in the matter. On the other hand, if it is treated as a purely definitional matter, it would certainly make it necessarily true but only at the cost of rendering it completely insignificant. In a certain sense, the word 'declarative' in the modern formulation does just this, for it is defined in terms of the properties of truth and falsity, though it does not explicitly specify that they are absolutely exclusive of each other.

The trouble with exclusion of properties is that, at least in respect to those which concern the empirical domain, it is not clear whether they really exclude each other or not. As Körner has argued, there can always be the possibility of a situation arising in the case of predicates applicable to the empirical realm where it is not clear as to which of the predicates applies. As both of the supposedly exclusive predicates seem to apply legitimately to the situation, one can only conclude that either both are applicable, or neither is. This has given rise to what has been called the logic of inexact predicates. However, if the inexactness is intrinsic to the predicates or to their applicability, one wonders how any logic could remove it.

The ontological problem raised by intrinsically inexact predicates has seldom been considered in detail by those who have tried to build a logic of inexact predicates. The problem relates to the question of whether such a situation is to be interpreted as indicating an intrinsic ambiguity or indeterminateness in reality itself or as an inalienable limitation or defect in the condition or constitution of human knowledge. Usually, such a dilemma has been felt in relation to the problems raised by certain facts in physics, both

10. On this and other related problems see the author's 'Law of Contradiction and Empirical Reality', *Mind*, April 1957.



at the microscopic and cosmic levels. But the issue is of wider import. Yet, except for Kalidas Bhattacharyya,<sup>11</sup> whose work is little known outside India, few have attempted to deal with it in a focal manner.

However, the ontological implications of the logic of classes centre even more round the notion of a class of all classes. The issues arising in this connection are well-known and relate to what in the literature on the subject has come to be called Russell's Paradox. The paradox is basically a paradox of self-reference, and the issue relates to the question of whether the notion of a genuine totality is capable of being intelligibly conceived without giving rise to self-contradiction. The consensus is that it cannot be so conceived, and the usual way out is to adopt some version or other of the Theory of Types. But as the theory prohibits in an ad hoc manner that any class may be a member of itself, it generates an unending series of classes of classes, each of an order higher than the other. In other words, it makes the notion of an achieved totality impossible in principle.

It may be contended that the ontological thrust of logic lies elsewhere. After all, the central crux of logic lies in the if-then relationship, for it only says that if one accepts something, then one has also to accept something else. Even with respect to truth-functionality we can only say that if one accepts a certain number of truth values which a sentence may have, and if one also accepts that certain connectives require a certain minimum number of sentences to form a new sentence, then it follows that there will be so many distinct truth-functional operators in the system. Logic, in this perspective, makes no assertions. Or rather, it only asserts hypothetical relationships. As the central logical contention is that one cannot simultaneously accept something and reject that which necessarily follows from it, the relation of implication becomes really basic to it. Thus, even though it is only one of the dyadic operators in a two-valued truth-functional logic, yet, in a sense, it is more fundamental than others.<sup>12</sup> Whether this

11. Kalidas Bhattacharyya, *Alternative Standpoints in Philosophy*, Calcutta, Dasgupta, 1953.

12. This point arose in a discussion with Dr. R.S. Bhatnagar of the Philosophy Department of the University of Rajasthan, Jaipur, and I am thankful to him for it.

fundamental character can be accepted and articulated while remaining within the boundaries of truth-functional logic, is difficult to say. Still, whether fundamental or not, the ontological commitment of logic may be thought of as lying in the assertion of the realities of relations, rather than of anything else.

If logic cannot be possible without assuming the reality of relations, then they would have to be provided with a reality, for logic would not like to commit suicide and argue for its own impossibility. As Russell argued long ago, the reality of relations, particularly those of the asymmetrical kind, implies a pluralistic world. *Ergo*, if logic implies the reality of relations and the reality of relations implies a pluralistic world, we might conclude that the reality of logic implies a pluralistic ontology. But is logic real? Bradley had argued, before Russell, for the unreality of relations, and long before him, the Buddhist philosopher Nagarjuna had argued for the unreality of anything which anybody held to be real. But, does everything include everything? Does it include itself, for example? If it does not, how can it talk of everything? If it does, surely we have, besides the usual paradoxes of self-reference, the peculiar problem of the unreality of that which proves the unreality of everything else? If it does not, we have the problem of the ontological reality of that process or argument which declares everything besides itself to be unreal.

The problem of the ontological presuppositions, implications, and status of logic is important as there has occurred in recent times a concerned and persistent attempt to develop a logic with no ontological commitments whatsoever. In a certain sense, the claim of logic to be true in all possible worlds had already implied it. Nagel had explicitly argued for the ontological neutrality of logic in his well-known book *Logic without Ontology*.<sup>13</sup> This could perhaps be understood as the implication of the essentially analytical view of logic which has been entertained by most logicians and philosophers. It would be interesting in this connection to ask about the position of those who do not subscribe to the analytic-synthetic dichotomy and, in fact, have attacked its very foundations. The position of Quine is well-known in this respect but it does not seem to have been noticed that once the distinction is

13. E. Nagel, *Logic without Metaphysics*, Glencoe, Illinois, 1957.



given up, it would be impossible for logic not to have ontological implications. Also, the so-called contingent character of any ontology will infect logic also, unless the converse infection is allowed to make ontology necessary.<sup>14</sup>

Now, logic does not seem to be about any entities whatsoever. Not even in the *prima facie* sense in which mathematics seems to be about numbers. If it is about anything, it is perhaps about sentences and the relationships between them; not about any *specific* sentences, but rather sentence forms of which particular sentences may be regarded as exemplifications. As for relationships, it is perhaps the one designated by the phrase 'following from' that may be said to be its specific concern. The ontological question with respect to logic would thus centre around the ontological status of sentences and those of sentence forms, and the relation of 'following from' which is supposed to hold between them.

What sort of entity is a sentence and what sort is the sentence-form with which logic is supposed to be concerned with? However one may conceive of sentences, it would be difficult to think of them as natural entities, and if they are not natural entities, sentence-forms could hardly be considered as such. If anything, they would have to be regarded as abstractions from a type of entity which itself could not be considered as 'natural' in any relevant sense of the term. Yet, sentences are the sort of things that are seen and heard, or even felt and touched as in the case of persons who are blind and use braille—but to the extent that they are only such, they cease to be sentences. To use an older terminology now out of fashion, visibility or audibility or even tactuality is not their essence, even though without them they could perhaps never be known.<sup>15</sup> The being of a sentence, so to say, lies completely outside itself—something which is hard to accept. Yet if there are no such things as sentences, there would perhaps be no such things as sentence-forms either, and it is difficult to see how there would be any logic if this were so.

14. For some aspects of this problem see the author's article 'The Synthetic *a priori*—Some Considerations', *Philosophy*, April 1961.

15. We say 'perhaps', for if telepathy were to be a fact, it would have to be admitted that one could 'know' what is conveyed without its being embodied in something which is apprehensible by the senses.

The desire to free logic of any ontological implications has resulted in the elaboration of logics which will be 'true' in all possible worlds, or perhaps even if there were to be no world. But it is difficult to understand what is exactly meant in this context. First, to the extent that logic is supposed to be analytically true, it is difficult to make sense of the idea that it has to be true of any world. Perhaps the analogy is with the notion of *application* in mathematics. If that is the analogy, then it also follows from the analogy that the application is something extra-logical which does not affect the logical validity of what is sought to be applied. Perhaps this is what is meant by the picturesque phrase 'true in all possible worlds'. But if this were so, then it would be necessarily so by definition, and the question of finding if it were actually true or not of all possible worlds would not arise.

On the other hand, what exactly is meant by a 'world' is not made clear. Does an object of sense experience count as a 'world'? What about the senses themselves? Are they to be considered as their own objects, or are they to be regarded as outside the 'world'? Whose sense experience is to constitute the world? Also, are all sense experiences to be given an equal status, or is a distinction to be made between them? If so, on what grounds? Do these grounds lie within the world or outside the world? Similarly, what is meant by the adjective 'possible' in this context? What would it mean for a world to be impossible? Since the distinction itself is usually drawn on grounds of logic, the notion of a 'possible world' comes to mean that which is not self-contradictory or in other words permitted by logic.

The so-called ontological problem is usually supposed to arise with respect to individual constants and how they are to be interpreted in case they are admitted in logic.<sup>16</sup> The problem becomes complicated by the issue of whether the inference rules concerning Universal Instantiation and Existential Generalization are to be allowed such constants. The first thing to note in this connection is that the problem arises only in relation to quantificational logic. It has no place in the sentential or the propositional calculus. The reason for this lies in the fact that the argument forms of quantificational logic depend on 'all' and 'some' which themselves are

16. The case arising in respect of classes has already been discussed.



ontological in character. 'Some' is interpreted in a sense which commits one to an empirical ontology, whether one wants it or not. As most of the logicians happen to be empirically inclined at present, they are happy with the interpretation and feel that it logically underwrites their empiricism. The 'all', on the other hand, is not only treated in such a way as to raise the problem of the ontological status of hypothetical entities and relationships, but also to give rise to the problem of counterfactual conditionals. Further, as we pointed out earlier, the 'all', in any empirical sense, is intrinsically incapable of being comprehended because of the simple fact that in any empirical perspective time has inevitably to be taken as real. If, therefore, logic is not to have any ontological implications, quantificational logic will have to be banished from the realm of logic.

This is generally masked by the fact that the usual interpretation of 'all' and 'some' is found in the realm of mathematics which does not seem infected by temporality. But the moment we leave that realm, ontological issues begin to crowd on every side. Mathematics, of course, raises its own ontological problems, especially those that concern the assumption of 'actual infinities'. But, however great the problems arising from this field, they do not arise from the nature of the quantifiers, especially when they refer to the empirical domain.

Further, the attempt to free logic of ontology thrives on a distinction between what logic is about and logic itself. However valid this distinction may be in limited contexts, it fails to make sense when the basic ontological issue is in question. If, for example, logic requires an indefinite supply of symbols, each distinguishable from the other, then an ontological pluralism has to be accepted if logic itself has to be. Similarly, as the symbols have to be apprehensible by the senses, not only has the reality of the senses to be admitted but also the reality of space in which the different instances of the same symbol may occur. This, of course, ignores the distinction between logic and meta-logic, but in any discussion of ontology it is bound to be so. Those who will like to stick to the distinction will have to accept an ontology of infinite hierarchies of 'meta-metas', with some of the same consequences we have earlier discussed in relation to the theory of types. The choice between the paradoxes of self-inclusion and the difficulties

of infinite regress may be left to the taste of individual philosophers. But the sleight of hand and the self-deception which lie behind the talk of an ontology-free logic need to be exposed so that the philosophically unwary are not taken in.



## LAW OF CONTRADICTION AND EMPIRICAL REALITY

It may be said to be a fairly accepted presupposition of both science and philosophy that contradictory qualities cannot hold true of the same existent in any meaningful sense of the term. Philosophers have sometimes gone even to the extent of asserting the unreality of the empirical world because of its supposed possession of contradictory qualities. But, by and large, the research for empirical or rational coherence has always assumed the foundational significance of the Law of Contradiction for any understanding on our part of the world of empirical reality. It shall be our task in this paper to show some of the puzzling implications of this assumption and to suggest some doubts about its so-called self-obvious character.

The Law of Contradiction is generally formulated as 'A cannot be both B and not-B' where A and B are unrestricted variables. There are no particular values of A or B for which alone the statement is supposed to hold true. In fact, if any such restriction were to be imposed upon the formal expression of the law, it would immediately lose its inevitable, self-obvious, necessary character and become merely an empirical statement true only in certain cases of its exemplification.

The empirical interpretation of the formal law, however, immediately meets with a difficulty. Any empirical determination of the variables A and B raises immediately the question concerning the interpretation of the variable not-B. If, supposing, we empirically interpret it to mean 'the table cannot be both red and not-red', then the question arises as to what exactly is meant by the term 'not-red'. Formally, the term 'not-B' is supposed to include all that is not B. In fact, it is generated through what is known as 'division by dichotomy' and which, just because of this formal all-inclusive character, can never be invalid. But if we

interpret the term 'not-red' in this sense we will have to say that 'the table cannot be both red and square', for 'being square' is included under the term 'not-red'. Such a consequence, however, would hardly be acceptable to any person, for it is so obviously false that people would rather give up the Law of Contradiction than accept such a consequence.

The only way out of the difficulty is to restrict the interpretation of the term 'not-B' to the determinates of the same determinable to which the term 'B' may be said to belong. If, for example, the term 'B' is interpreted to mean 'red', then 'not-B' includes only other determinates of the determinable of colour and not everything which is formally not B. To put it in other words, the counter-domain of B is empirical and not logical.

Such a solution, though generally accepted, is not without difficulties of its own. The term 'not-B' interpreted, say, as green gives us merely the contrary and not the contradictory of B as was supposed to be done in the formal presentation of the law. The self-obvious necessity of the law derived, however, from the contradictory relationship of B and not-B which is absent when the interpreted variables are related as mere contraries. It is, of course, possible to interpret the formal presentation in such a way as to be compatible with both a contrary and a contradictory interpretation. The law merely states that 'A cannot be both B and not-B'. Now, it is true of the contraries that they cannot both be true, though they may both be false. The term B and not-B can be interpreted as contradictories only when we take the Law of Excluded Middle also into account. It is only in conjunction with the statement 'A must either be B or not-B' that the relation between B and not-B can be treated as one of contradiction. But on such an interpretation, the term 'not-B' would not have an unrestricted domain limited only by the domain of the term 'B'. It will have to be restricted in some further sense, but whether such a sense can be found in pure formal logic, is difficult to say. In fact, on the formal plane, the Law of Contradiction cannot be torn apart from the Law of Excluded Middle and the terms B and not-B can never be treated as contraries but only as contradictories. It is only when we attempt an empirical or transcendental interpretation that the possibility of treating them as contraries and thus of giving up the Law of Excluded Middle can



arise. Brouwer, for example, has been able to deny the validity of the Law of Excluded Middle in mathematics only by treating it as concerned with some sort of entities which may be considered as transcendental in character. In contrast, Hilbert, who treats mathematics as a branch of or as identical with pure logic, cannot but treat B and not-B as contradictories implying, thus, the inevitable conjunction of the Law of Contradiction with the Law of Excluded Middle.

The question of the inevitable conjunction of the Law of Excluded Middle with the Law of Contradiction may, however, be raised in another way, as suggested by Om Prakash. Instead of asking whether the Law of Contradiction necessarily involves the Law of Excluded Middle, we might ask whether, on the formal plane, the Law of Excluded Middle necessarily involves the Law of Contradiction. In other words, does 'A must either be B or not-B' involve that 'A cannot be both B and not-B'? *Prima facie*, it seems that it must be so. But if we look closely, we shall find that the Law of Excluded Middle is, in fact, a disjunctive proposition from which, according to the rules of disjunctive inference, the conclusion "A cannot be both B and not-B" cannot follow. It should be recalled that, according to the rules of disjunctive inference, by affirming either of the alternatives, we cannot deny the other alternative, though by denying one of the alternatives, we can always affirm the other. If, for example, we say in the major premise 'A is either B or C' and in the minor premise 'A is B', we cannot say in the conclusion "A is not C". The disjunctive proposition is *formally* supposed merely to assert that A is characterized, *at least*, by one of the alternatives but does in no case *exclude* the possibility of its being characterized by *both*. The disjunctive proposition 'A must either be B or not-B', therefore, distinctly allows for the possibility that 'A can be both B and not-B'. Unless, therefore, we are prepared to modify the rules of disjunctive inference, we must accept this consequence even if it appears as a manifest absurdity.

It may be suggested that the disjunctive proposition is generally an assertory proposition while the Law of Excluded Middle is formulated as a necessary proposition. It is the force of the 'must' which suggests that A cannot be both B and not-B. But in logic, it is the *form* of the proposition that determines the permissible

consequences which can be drawn and, as far as I am aware, no such limitation on the disjunctive form of 'Either-Or' has yet been made.

Pratap Mehra has suggested another reason why on the assertion of the alternative 'A is B' in the minor premise we cannot deny 'A is not-B' in the conclusion. The conclusion, it is agreed, must not contain any proposition which has already occurred as a premise. Now if the major premise is 'Either A must be B or not-B' and the minor premise 'A is B', then the conclusion cannot be 'A is not not-B' for it is just equivalent to 'A is B'. The same will be true if we assert 'A is not-B' in the minor premise. 'A is not B' in the conclusion would be equivalent to 'A is not-B' in the minor premise.

This reason, however, is applicable only because of the nature of the alternatives B and not-B. The rule of the disjunctive syllogism is, on the other hand, more general and disallows the negation of the other alternative on the assertion of either on ground of the implicational possibilities ascribed to the pure form designated by 'Either-Or'.

Thus, while the Law of Contradiction seems necessarily to involve, on the formal plane, the Law of Excluded Middle, the latter cannot involve the former unless we are prepared to modify the rules of disjunctive inference in general. On any empirical or transcendental interpretation, on the other hand, neither need involve the other.

The difficulty arising from the behaviour of 'not-B' as a contrary rather than a contradictory may, however, be met in another way.

It may be suggested that while the term 'not-B' certainly does not include all that is formally not B, still it functions as the contradictory and not the contrary of B. This function it may be said to perform by virtue of the fact that it includes under itself not some particular determinate or set of determinates but rather *all* the determinates of the relevant determinable excluding, of course, the one denoted by the term B. If, for example, the term B is interpreted to mean 'red', then the term not-B includes *all* the determinates of the determinable of colour excluding only the 'red' denoted by the term B. The collective exhaustion of all the determinates between B and not-B would, then, be the



ground of their being treated as *actual* contradictories even if they may not be formally so. But such a treatment can only be valid if it is assumed that A must necessarily be coloured for, if it is possible for A not to be coloured then it may neither be B nor not-B. This will only be possible if we treat white and dark as colours, for then it would be possible to say that A, if it is a physical object, must necessarily be coloured. However, even then, the necessity will only be empirical and not logical, for it is certainly not self-contradictory to say that 'A is not coloured'. Unless the statement 'A is coloured' is taken to be an analytic statement, the B and not-B determinates of the determinable of colour cannot be taken as logical contradictories.

It should further be noted that it is on no *formal* ground that we say that an empirical object can be characterized by determinates of different determinables. If a table can be both square and red, the reason for this is not something formal but only experiential. Similarly, we suggest that if a table cannot be both red and green it is not because of any formal ground but only because experience shows it to be so. The formal Law of Contradiction, as we have just seen, can neither warrant the former nor support the latter.

However, even if we grant that the terms B and not-B should be confined to the determinates of the same determinable and that each physical existent must be characterized by some determinate of every determinable, it does not follow that the Law of Contradiction can safely be applied to empirical reality without any further trouble. The formal scheme 'A cannot be both B and not-B' interpreted, for example, as 'The table cannot be both red and green' can easily be shown to be wrong by pointing to a table whose one-half is painted red and the other-half green. It is bound to be objected that the table must be red and green *at the same point* and not in different regions of its surface. But the objection, though usual and familiar, is seriously mistaken. What was maintained in the interpreted proposition was that 'The table cannot be both red and green' and not that the same point of the surface of the table cannot be both red and green. Obviously a point on the surface of the table is not the table and the statement that we made was about the latter and not the former. There seem, then, only two alternatives: Either we deny the

necessary truth of the empirical interpretation of the Law of Contradiction with respect to physical objects or restrict the interpretation of the variable A to 'same points on the physical surfaces of objects'.

As the former alternative can hardly be chosen without involving a radical reorientation in our basic presuppositions of science and philosophy, the latter alternative is bound to be chosen. But even the drastic restriction on the interpretation of the variable A can hardly get us out of difficulties. The statement that 'the same points on the physical surfaces of objects cannot be both red and green' can easily be shown to be false by pointing to a uniform stretch of surface of some object which is at one time green and at the other time red. The interpretation of the variable A must, thus, be further restricted to 'the same points at the same moments of time' which, then, according to the law of contradiction, cannot be said to be characterized by the different determinates of the same determinable.

This is hardly new. It has always been admitted that a thing cannot possess contradictory qualities at the same space and at the same time of its existence. But it has generally been overlooked that such an admission raises formidable difficulties of its own and makes the *a priori* applicability of the Law of Contradiction far from self-evident, as is generally assumed in most circles.

The first question that arises concerns the empirical meaningfulness of 'the same point-instant' restriction on the interpretation of the variable A. If the term 'point-instant' is something empirical then, obviously, it cannot be the theoretical limit of a possible infinite division but something as immediately observable as the ordinary objects or things of the world. But in such a case if it is pointed out that such a 'point-instant' is *actually* characterized by different determinates of the same determinable then it cannot be argued that the very possession of these different determinates proves that it could not either be the *same* point or the *same* instant which was so characterized. If someone were still to argue in the above manner, it would only show that the person was not using the words in any empirical sense at all. In fact, he could hardly be refuted, for each time someone showed him such a case he could get away by saying that it could



not have been the same point or the same instant of time. But he would have achieved this seeming invincibility by surrendering the basic fact about the inevitable empirical applicability of the Law of Contradiction, viz., the empirical interpretation of the variable A. If, for the truth of the Law of Contradiction, A must be interpreted in a non-empirical manner then the whole attempt to show its inevitable empirical application is given up at the very foundations.

The points, then, must be physical and not mathematical if we are to talk significantly about empirical objects at all. But if the points are physical, there seems no self-evidence about the supposed necessity of every such point being characterized by only one determinate of a particular determinable. Further, it will be extremely difficult to determine what exactly is meant by the term 'physical point'. If it is supposed to refer to the fundamental particles believed to be indivisible by the science of physics, then it would vary with the relevant developments in that science. If, however, field phenomena are considered to be more fundamental, then the concept of 'physical points' can only be considered in Whitehead's phrase as a 'fallacy of simple location'. Perhaps, Planck's basic quantum of action may provide that indivisible invariant which the notion of physical points requires. But whether it would suitably fulfil the function for which it is required, is more than we can say.

Even if a satisfactory correlate of the notion of physical points can be found in the science of physics, it would, we think, not serve the purpose. The Law of Contradiction would, then, apply *only* to sub-atomic particles and not to atoms, molecules or other microscopic or macroscopic objects. But, in that case, the law will not be applicable to most objects and, though certainly empirical at one level of its application, it would mostly be irrelevant for man's concern with the macroscopic world of his sense experience. In fact, for the purposes of the Law of Contradiction, the object is atomistic in character—a view that does grave injustice to the unity of the object. A table, for example, is a unitary object and not merely a sum total of the so-called physical points in the universe.

In whatever manner we may solve the difficulties resulting from the notion of the 'same point', those resulting from the co-

ordinate concept of the 'same instant' will still remain. The question about the logical *versus* the physical interpretation and the difficulties concerning the latter will be the same as in the former case. There will, on the other hand, be the added difficulty of defining 'simultaneity' for different observers.<sup>1</sup>

Further, the concept of the 'same instant', whether interpreted logically or physically, would make the application of the Law of Contradiction to empirical reality either impossible or irrelevant. If it is *only* at the *same instant of time* that A cannot be both B and not-B, then if A *endures in time*, the law cannot apply to it and if it persists only for an instant, it is irrelevant to it. If an empirical object is considered as a patterned unity enduring through multitudinous instants of time, then it is impossible that the law can apply to it. On the other hand, if the thing itself is considered as instantaneous or momentary, the law, though true, is irrelevant—for, the thing at every different moment is a different thing.

It has not been noticed by thinkers that the overriding phrase 'at the same point of space and at the same instant of time' involves a Humean or Buddhist view of empirical reality which cuts at the very root of the significant applicability of the Law of Contradiction to that realm. For, if a thing is no thing but merely a point-instant series of events then it surely can be characterized by B and not-B at different instants of time. If it is suggested that the thing at a different instant of time is a different thing, the situation would be no better. In that case, it will be a tautology to say that the Law of Contradiction necessarily applies to empirical reality. The statement, while true, will be of no significance, for everything would last only for an instant and then, for purposes of contradiction, would be no more.

The point-instant limitation on the interpretation of the variable A leads to another serious difficulty if it is interpreted in the rigorous mathematical manner as it should be to make the appli-

1. I am not quite sure whether the same corresponding difficulty will not arise with respect to the operational meaning of the concept 'same point' for different observers. It is, of course, possible that a corresponding difficulty may arise with respect to the operational meaning of the concept 'same point' for different observers. But I am not quite sure about it.



cability both self-evident and necessary. It is well-known, following Cantor's discovery of the transfinite numbers, that the number of points on a surface, however large or small, is the same. Now if a thing must be treated, for the purposes of the Law of Contradiction, as consisting of nothing but points, it follows that all things are empirically the same. This, however, is difficult to accept, for it is equally well accepted that any argument which leads to infinite consequences concerning physical objects is invalid. In Ryle's terminology, we are harnessing the wrong horse to the wrong carriage. But this is just our contention that the empirical applicability of the Law of Contradiction requires us to perform this impossible feat. Infinite analysis in the field of physical objects is bound to lead to paradoxical results, but this is just what is demanded if the Law of Contradiction is to apply to empirical reality. The dilemma can be avoided only by giving up the demand, but that would involve, in its own turn, new difficulties.

The empirical interpretation in terms of 'the same point of space' leads to another difficulty with respect to those empirical objects which cannot, possibly, be spatially characterized. Mental phenomena, it is more or less agreed, cannot be characterized with respect to their spatial co-ordinates. Even if they are considered to possess some extensity or spread-outness, it would be difficult to regard them as located in space in any ordinary sense of the term. If, then, the point-instant interpretation of the variable A is held necessary, then the Law of Contradiction cannot apply to, at least, one realm of empirical reality, viz., the mental phenomena. A possible alternative will be to limit the necessity to the 'instant' interpretation only. But, then, at the same instant B and not-B could characterize the same thing at different points and thus invalidate the Law of Contradiction. Either, then, we have to deny the applicability of the Law of Contradiction to mental phenomena which are undoubtedly empirical or we must maintain that they can be as much characterized by spatial determinations as any other physical object.

The common assumption that the Law of Contradiction must necessarily apply to empirical reality seems, thus, to be so full of puzzling features that it appears surprising how it could have been accepted as self-evident by so many thinkers in the field of

philosophy. Far from being self-evident, it reveals, as we have seen, problems both of a formal and an empirical nature. At the formal level, the problems centre around the interpretation of the variable not-B and the relations between B and not-B. At the empirical level, the issues centre round the interpretation of the variable A. The 'point-instant' interpretation without which the empirical applicability of the law cannot be defended makes, in its own turn, the application either impossible or irrelevant. The cumulative effect of all these difficulties is, we suggest, to make impossible the application of this law. Either there is something radically wrong with the arguments we have given or this age-old self-evident axiom of science and philosophy is only a hollow myth. There is no other alternative, and unless we are shown that the former alternative is the correct one, we cannot but choose to assert the latter.



## SYMMETRY, TRANSITIVITY AND REFLEXIVITY

It is commonly assumed that relations which are both symmetrical and transitive are also necessarily reflexive in character. Russell, for example, writes: 'It is obvious that a relation which is symmetrical and transitive must be reflexive throughout its domain.'<sup>1</sup> The obviousness of the necessity may perhaps be not so 'obvious' after all. At least, Russell has not here given any reason to make us accept the 'obviousness' except one which we shall discuss later.

A supposedly clinching logical proof can, however, be given for this 'obviousness'. Given symmetry, if  $xRy$  then  $yRx$ , and given transitivity, if  $xRy$  and  $yRx$  then  $xRx$ , which is the symbolic characterization of reflexivity itself. Thus, reflexivity is seen necessarily to follow from symmetry and transitivity taken together.

The proof seems logically unexceptionable, but if we consider certain concrete examples we are seemingly led into difficulties. Take, for example, the proposition: *John is different from Tom and Tom is different from Harry*. It seems clear that if John is different from Tom, Tom must also be different from John and if John is different from Tom and Tom is different from Harry, John must also be different from Harry. The two characteristics of symmetry and transitivity are, thus, satisfied in this case. But obviously the relation does not thereby become reflexive, for it certainly is not true that John is different from John.

It may perhaps seem difficult to dispute that, at least in this particular instance, symmetry and transitivity do not lead to reflexivity. Yet, the formal proof that it must be so, seemed completely valid. Where, then, is the clue to this seeming incompatibility between the concrete instance and the logical proof?

1. *Introduction to Mathematical Philosophy*, p. 16.

Logicians have sought to solve the difficulty by denying the transitivity of the relation of 'difference'. It is contended that if  $x$  is different from  $y$  and  $y$  is different from  $z$ , then  $x$  need not be different from  $z$ , for  $x$  and  $z$  may be identical. Russell, for example, in denying the transitivity of the relation 'greater or less' among numbers gives the reason that 'if  $x$  is greater or less than  $y$ , and  $y$  is greater or less than  $z$ , it may happen that  $x$  and  $z$  are the same number'.<sup>2</sup> Similarly, he has sought to deny the transitivity of the relation 'brother or sister of' which ordinarily would be regarded as both symmetrical and transitive and yet irreflexive, on the ground that 'if  $x$  is brother or sister of  $y$ , and  $y$  of  $z$ ,  $x$  may not be brother or sister of  $z$ , since  $x$  and  $z$  may be the same person'.<sup>3</sup>

This is the only reason, to my knowledge, that has ever been given by Russell or anybody else for denying the transitivity of these relations which, though symmetrical, are yet irreflexive in character.<sup>4</sup> It should be noted that the symmetrical character of these relations has never been questioned and that the argument for denying their transitivity *has nothing to do* with the sym-

2. *Ibid.*, p. 33.

3. *Ibid.*, p. 31. Italics mine.

4. It may be questioned by some whether the *only* reason for denying the transitivity of such relations as that of 'difference' consists in the possible *identity* of  $x$  and  $z$  as suggested by Russell.  $x$  and  $z$  may not be identical and yet if  $x$  is different from  $y$  and  $y$  is different from  $z$ ,  $x$  may not be different from  $z$ , for  $x$  and  $z$  may be *similar*. It is the possible *similarity* and not the possible *identity* of  $x$  and  $z$  that makes the relation intransitive. Thus, unless we choose to treat bare numerical difference as 'being different', we cannot find the *sole* reason for denying the transitivity of such a relation in the Russellian argument.

*Prima facie*, the objection seems pretty strong. But, if closely examined, it would appear to thrive on an ambiguity. 'Similarity' is a tricky word. If two things are 'similar', that does not mean they are not 'different', but *only* that the points of difference are not considered as relevant or important by us. With a slight change of context, interest, or purpose, the 'differences' may loom larger and become more important than the 'similarities'. Two mango trees,  $x$  and  $z$ , for example, are similar and each different from the banyan tree  $y$ . But this does not mean that the trees  $x$  and  $z$  cannot be different in, for example, the taste of their fruits or the girth of their trunks or the width of their leaves or the abundance of their harvest or the time of their ripening, etc. We can, of course, think of examples where  $x$  and  $z$  have complete *qualitative identity* with



metrical, asymmetrical or non-symmetrical character of these relations. The argument, it has not been noticed, if valid, proves too much. It rules out the transitivity of all relations which are irreflexive, independent of the question whether they are symmetrical or asymmetrical in their nature. For example, if *x* precedes *y*, and *y* precedes *z*, *x* may not precede *z*, since *x* and *z* may be the same thing. This appears so obviously correct that it seems amazing to find Russell writing on the same page 'if *x* precedes *y* and *y* precedes *z*, *x* must precede *z*'.<sup>5</sup> Why must *x* precede *z* if *z* may refer to the same thing as *x* and *x* cannot precede itself? Russell writes further: 'This may be illustrated by the same instances as before: *less*, *earlier*, *left of*'.<sup>6</sup> Here 'less' is given as an example of a transitive relation, while just two pages after, on p. 33, the relation 'greater or less' is considered as not transitive, 'for if *x* is greater or less than *y*, and *y* is greater or less than *z*, it may happen that *x* and *z* are the same number'.

*Prima facie*, no relevant difference seems to be made to the argument by the substitution of 'greater or less' for 'less' to make the one intransitive and the other transitive.<sup>7</sup> The argument for the intransitivity of the relation 'less than' can almost be put in the same words: 'for if *x* is less than *y*, and *y* is less than *z*, it

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just numerical diversity. Two *equal* specimens of an element with identical physical and chemical properties may be said to exemplify such a situation. Here *x* and *z* could not be said to be different from each other in any respect, even though they may each be different from another element *y*. But this is only because we have assumed *complete qualitative and quantitative identity* between them and *decided to ignore* the only difference between them, i.e., numerical diversity. This, obviously, comes to the same thing as the Russellian argument from the identity of *x* and *z*. Thus, either the 'similarity' between *x* and *z* does not exclude the 'difference' between them or it reduces itself to the Russellian identity with the postulation of complete identity and the decision to ignore numerical diversity.

5. *Ibid.*, p. 31. Italics mine.

6. *Ibid.*, p. 31. Italics author's.

7. The relation "greater or less" may be interpreted in a manner different from the one in which Russell has understood it here and in that case the usual Russellian argument for the denial of transitivity would not be the sole ground for its denial here. If, for example, "greater or less" is understood as, say, "taller or shorter" then obviously the reason for the denial of its transitivity would not lie in the possible identity of *x* and *z*. A tree

may happen that *x* and *z* are the same number'. Russell would, of course, deny the very *possibility* of putting the argument in such a way. It cannot even possibly happen that *x* and *z* are the same number, for such a possibility is excluded by the asymmetrical character of the relation 'less than'. If *x* is less than *y*, and *y* is less than *z*, *z* cannot be the same as *x*, for if *x* is less than *y*, *y* cannot be less than *x*. The asymmetrical character of the relation does not permit the identification of *x* and *z* on which the argument was supposed to rest. In the usual Russellian argument, it is the symmetrical character of the relation that permits the identification of *x* and *z*, though he has nowhere explicitly stated it to be so. It is only on this supposition that some sense can be made out of his denial of transitivity to 'greater or less' and his accepting it in the case of the relation 'less than'.

Unfortunately for the argument, it is not so much grounded in symmetry as in the possible identity of the referents of different symbols, an identity that does not make the relation between them invalid. Symmetry itself, for example can be questioned on this ground. If *x* is 'greater or less than' *y* then *y* may not be greater or less than' *x*, for *y* may refer to the same number as *x* and *x* cannot be greater or less than itself. Or, to take another example, if *x* is the brother or sister of *y*, then *y* may not be the brother or sister of *x*, for *y* may be the same as *x*. The obvious reply would be that if *x* and *y* refer to the same person, *x* cannot be the brother or sister of *y*, since *x* cannot be the brother or sister of itself. But that is just what may be contended. One may argue that unless a relation is reflexive, it cannot be symmetrical, since in the exceptional event of *y* being interpreted to be the same as *x*, it will not hold good. Not merely this, even the statement of an irreflexive relation would become logically im-

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may be "taller or shorter" than a building and a building "taller or shorter" than a telegraph pole without it necessarily following that the tree must be "taller or shorter" than the telegraph-pole, since the two may be of the same height.

Such an interpretation, however, would remove it from our field of discussion altogether. We are interested *only* in those relations the *sole* ground of whose intransitivity lies in the possible identity of *x* and *z*, for we are certainly not denying that there are such things as intransitive relations.



possible, unless we introduce the limitation that *y* is not to be interpreted as *x*. Once, however, this limitation is accepted it would apply equally to transitivity and invalidate the argument given by Russell above.

It should be noted here that the reflexivity or irreflexivity of a relation is not so much a matter of logic as of intuitive judgment. Whether *x* can be greater than itself is not a matter of argument. Similarly, it may be contended that the transitivity of a relation is not a matter of argument, but of specific judgment. If *x* is greater than *y*, and *y* is greater than *z*, then the question of *x* being greater than *z* cannot be a subject of argument. For, if once the necessity of argument is admitted, it would have to be applied in the case of reflexivity also, where, perhaps, it cannot be applied at all.

The dilemma, by now, should be fairly obvious. Either we accept that a relation which is irreflexive can neither be symmetrical nor transitive, or we agree to the limitation on the interpretation of *y* or *z* and, thus, give up the position that a relation which is both symmetrical and transitive must be reflexive in character.

If we choose to deny transitivity to irreflexive relations, as we must if we do not accept the limitation on the interpretation of the symbols *y*, *z*, etc., we shall find it impossible to have an ordered series which can only be formed out of relations that are asymmetrical, transitive and connected. As asymmetry necessarily involves irreflexivity and as irreflexivity would, on the usual Russellian argument, necessarily involve intransitivity, the study of ordered series would become logically impossible.

Such an alternative will hardly be welcome to anybody. The acceptance of the limitation that a symbol is not to be interpreted in the same way as another symbol, unless allowed otherwise, would hardly disturb anything except the belief that symmetry and transitivity necessarily involve reflexivity.

It may be argued, however, that what we are asked to give up is not some idiosyncratic psychological belief but a strictly proved logical deduction. And as logicians we cannot give up a strictly deduced logical entailment even if it leads to empirical or non-empirical difficulties elsewhere.

But even though it is true that there are strict proofs to show that

reflexivity is entailed by symmetry, and transitivity,<sup>8</sup> yet all these proofs assume the possible identity of the third and the first term of a transitive relation. If such an assumption is given up, the proof will not be possible and thus the incompatibility between the logical proof and the concrete example would have been removed. Otherwise, no relation which is irreflexive would be symmetrical or transitive and even the symbolical transcription of an asymmetrical relation would become a logical impossibility.

8. A simple proof for such an entailment has already been given. A more complicated proof can be found in Quine's *Methods of Logic*, pp. 179-80.



## SYMMETRY, TRANSITIVITY AND REFLEXIVITY—SOME COMMENTS\*

S. BHATTACHARYYA

Daya in his paper 'Symmetry, Transitivity and Reflexivity' (*Analysis*, 19.1, pp. 7-11) has tried to prove against Russell that symmetry and transitivity of a relation do not necessarily imply its reflexivity. Here we shall try to show that he has failed in his attempt, for (i) none of his criticisms of Russell's argument is justified, and (ii) the solution he recommends is no better than Russell's.

Daya begins by citing a 'concrete' example: 'John is different from Tom and Tom is different from Harry', where he thinks 'symmetry and transitivity do not lead to reflexivity' (p. 8). To prove the symmetry of difference in this case he says 'It seems clear that if John is different from Tom, Tom must also be different from John' (p. 7); and to prove transitivity he asserts 'and if John is different from Tom and Tom is different from Harry, John *must* also be different from Harry' (pp. 7-8, italics mine). But we may ask here: Must John be different from Harry in this case? What if 'John' and 'Harry' be names of the same person? 'John must be different from Harry' does not follow from what he has stated, but follows from an unstated assumption that the different names, 'John' and 'Harry', must stand for different individuals. Thus it cannot be insisted that 'at least in this particular instance, symmetry and transitivity do not lead to reflexivity' (p. 8), for, even in this particular instance, difference has not been shown to be transitive.

Daya of course is aware of this objection, and he tries to meet it by challenging the validity of Russell's argument to deny the

\*Prof. S. Bhattacharyya's comments on my paper (No. 3) are published here as otherwise my reply, to him (No. 5) will not be intelligible to the reader.

transitivity of difference and other similar relations, and asserting that 'the argument for denying their transitivity *has nothing to do* with the symmetrical, asymmetrical or non-symmetrical character of these relations' (p. 9). This statement, which he simply asserts but does not prove, is wrong, for it is wholly on the basis of their symmetry that these relations are asserted to be intransitive. Daya himself acknowledges this when he says 'In the usual Russellian argument, it is the symmetrical character of the relation that permits the identification of x and z' (p. 10). Thus it seems that the most important argument of Daya against Russell is his argument against this identification, which we shall now discuss.

Daya thinks that if the possible identity of x and z is regarded as an argument against the transitivity of difference in 'x is different from y, and y is different from z', then 'symmetry itself... can be questioned', and 'even the statement of an irreflexive relation would become logically impossible' (p. 10). But this difficulty springs from his assumption that there are only two alternatives, either (i) to allow different variables to have the same value in all cases, or (ii) not to allow this in any case. This seems so obvious to him that he does not stop to examine the definitions of symmetry, transitivity, reflexivity, etc., and rashly asserts that the possible identity of x and y in 'x is "greater or less than" y' invalidates the symmetry of the relation 'greater or less than' (p. 10). But that he is wrong can be easily seen. A symmetrical relation, R, between x and y is defined thus:

$$(x) (y) (xRy = yRx)$$

Now it is obvious that this definition is equivalent to:

$$(x) (y) (xRy \supset :yRx.x = y \vee x \neq y)$$

which again is equivalent to:

$$(x) (y) (xRy \supset .yRx.x = y : \vee xRy \supset .yRx.x \neq y)$$

This shows clearly why even though the relation 'greater or less than' cannot hold between x and y when they are identical, its symmetry is not affected. There is no difficulty either in 'stating



an irreflexive relation'. An irreflexive relation, R between x and y can be defined thus:

$$(x) (y) (xRy \supset x \neq y)$$

Now the real nature of Daya's objection becomes clear. I have stipulated in the definition of irreflexivity that the different variables 'x' and 'y' cannot have the same value, whereas in the definition of symmetry (and, as can be easily shown, in that of transitivity) I have allowed them to have it. Apparently Daya does not want to allow this. Without stating anywhere what objection he has to my following this procedure, he asserts 'Once, however, this limitation is accepted it would apply *equally* to transitivity and invalidate the argument given by Russell...' (p. 10 *Italics mine*). If I do not apply the limitation 'equally Daya will perhaps say that I am inconsistent. But that there is no inconsistency can be easily shown by formulating explicitly the rule of substitution followed by Russell (and by logicians generally) thus: Different variables may have the same value unless otherwise stated. Though from Daya's insistence on applying the limitation 'equally', one would expect him to accept the rule 'different variables can never have the same value', what he actually says is very different. He recommends 'the acceptance of the limitation that a symbol is not to be interpreted the same as another symbol, *unless allowed otherwise...*' (p. 11. *Italics mine*). The qualifying phrase 'unless allowed otherwise' is to be noted carefully, for here he is withdrawing his earlier demand that the limitation be applied 'equally'. Now if it is withdrawn, as it apparently is, then the *only* objection to Russell's argument is also withdrawn. For it is clear that Daya cannot disallow exceptions to Russell's rule of substitution while allowing them in his own case. Moreover, he is definitely mistaken when he says (without proof) that the acceptance of his rule 'would hardly disturb anything except the belief that symmetry and transitivity necessarily involve reflexivity' (p. 11). His mistake will be apparent if we consider his 'concrete' example: 'John is different from Tom and Tom is different from Harry'. He thinks that he can now have the conclusion 'John must be different from Harry', for Russell's objection that 'John' and 'Harry' may be names of the same person cannot hold good because of the new rule of substitution. But now the difficulty is that 'John

must be different from Harry becomes analytically true, and, therefore, follows from any and every sentence. For example, I shall argue correctly, according to Daya's rule, if I say: 'John is identical with Tom, and Tom is identical with Harry; therefore, John is different from Harry.' Here the two sentences, 'John is identical with Tom' and 'Tom is identical with Harry' will be meaningless, because they violate Daya's rule; still they will imply the (analytic) truth 'John is different from Harry'. As a matter of fact the acceptance of his rule will necessarily lead to a total change of the ordinary system of logic, for

$$(x) (y) (x \neq y)$$

becomes a theorem. Daya has not realized that the rejection of one logically valid argument is impossible without rejecting a substantial part, sometimes the whole, of the system of logic. If, on the other hand, he appeals to his qualifying phrase, 'unless allowed otherwise', and add "John" and "Harry" may be names of the same man' to 'John is different from Tom, and Tom is different from Harry', then he cannot have the conclusion 'John is different from Harry', for he has come back to Russell's position. But though by frequently appealing to his qualifying phrase, he can avoid disaster, his rule will be disadvantageous to work with. For the cases where we want the different variables to be allowed to have the same value far out-number the cases where we do not, and Daya's rule that different variables have different values unless allowed otherwise, will mean constant addition of the clause that the variables are so allowed. Thus we find that Daya has not only failed to find any defect in the ordinary rule, but his own rule is much inferior to that which it replaces.

Now we can sum up. If exceptions are to be allowed, then Russell's rule has a technical advantage over Daya's rule. If exceptions are not to be allowed (for whatever reasons) then Daya's rule is as defective as Russell's. Thus Daya's polemic against Russell means simply that he wants to follow another rule of substitution and reject the usual one. He is, of course, free to follow his rule which, though practically disadvantageous, is still not self-contradictory, but he has not succeeded in showing why the ordinary rule has to be given up.



## SYMMETRY, TRANSITIVITY AND REFLEXIVITY

S. Bhattacharyya in his discussion<sup>1</sup> of my paper 'Symmetry, Transitivity and Reflexivity'<sup>2</sup> has raised some issues which perhaps could bear further discussion.

Leaving aside the minor points which I shall discuss in the latter part of this paper, the major issue seems to be about the rule of substitution which, however formulated, requires exceptions to be made in certain cases. Whether we say 'Different variables may have the same values unless otherwise stated', or 'A symbol is not to be interpreted the same as another symbol, unless allowed otherwise', the qualifying clause has to be added in order to safeguard cases where we want it to be otherwise. In such a situation where either of the formulations could suffice for our purposes, the choice is obviously dictated by the principle of economy, which would recommend the adoption of the rule which has the least exceptions. And Bhattacharyya is most probably right when he says 'the cases where we want the different variables to be allowed to have the same value far outnumber the cases where we do not' (p. 96), thus suggesting the adoption of the rule 'Different variables may have the same value unless otherwise stated' on grounds of pure economy.

A different formulation, however, sets our minds in a different direction. It makes us ask questions which we would not otherwise ask. It is the exceptional case that has to be justified, argued about, given reasons for. And as the exceptions are different on different formulations, the mind asks justification of different things in different cases. But the crucial question is, 'Why do we want the exception to be made in certain cases?' The obvious answer is 'because the observance of the rule would lead us into con-

1. *Analysis*, vol. 19, no. 4.

2. *Analysis*, vol. 19, no. 1.

tradictions and difficulties'. The generalized rule of substitution, however formulated, would then merely mean that different variables may have the same value if the truth-value of the statement remains unaffected by such a substitution. In case it is so affected, the different variables cannot have the same value in the statement concerned. The main question then is, 'What happens to the truth-value of a statement, if different variables are allowed to have the same value in that statement?'

The question, I think, is important. For, if a statement becomes false or a relationship untrue *only* in the case where different variables are given the same value, there seems a *prima facie* strong case for making exception to the rule. Otherwise, the exception will never be made. Now everybody, including Bhattacharyya, seems agreed that exceptions have to be made. But nobody, as far as I am aware, has explicitly formulated the criterion for deciding when an exception is to be made. I suggest the above as a fair formulation of such a criterion.

All the concrete instances which I had given in my paper (such as 'being different from' or 'being brother or sister of') failed to be transitive *only* in the case when the variable  $x$  and  $z$  were given the same value. This, I suggest, is a clear case for introducing the exception and treating these relations as *transitive*, for their transitivity fails *only* in one case, viz., when  $x$  and  $z$  have the *same* value.

Normally, the rule for transitivity does not require that the relation be valid even when the variable  $x$  and  $z$  have the same value. The formal presentation merely states:

$$(x) (y) (z) \left\{ (xRy) \cdot (yRz) \supset (xRz) \right\}$$

It is valid *even if* we postulate the condition  $x \neq z$ . Mr. Bhattacharyya's formulation of the rule of substitution for symmetry takes the form:  $x = yvx \neq y$ . Similarly, in the case of transitivity it could, as he suggests, be formulated as:  $x = yvx \neq yvx = zvx \neq zvy = zvy \neq z$ .

This, it should be obvious, does not require that different variables *have* the same value, but only that they *may* have it. In case the transitivity fails for any of the identities (e.g.  $x = z$ , the case in which we are interested) the relation cannot be said to be



not transitive. The transitivity of a relation can, on such a formulation, be denied *only* if it fails to hold in both the cases, i.e., when  $x = z$  and when  $x \neq z$ . On a purely formal ground, then, a relation cannot be denied transitivity if it *only* fails to hold in the situation where  $x = z$  unless we make this a *necessary* condition for asserting transitivity of any relation whatsoever. But once we make this a necessary condition, many of the relations usually considered to be transitive will cease to be so. This is all that I wanted to convey in the concrete examples I gave against Russell that once the *formal* requirement of  $x = z$  is considered *necessary* for transitivity, the relations which Russell is holding to be transitive will have to be considered otherwise.

It may be suggested that the formal requirement of  $x$  being the same as  $z$  becomes necessary only when the relation is symmetrical. But is it really so? From where does this presumed necessity flow? Neither Russell nor Bhattacharyya seems to have tackled this question. On a purely formal plane, the substitution rules for both symmetry and transitivity are given in terms of a disjunction whose generalized form may be symbolized as  $x = y \vee x \neq y$ ,  $x$  and  $y$  standing for any variables whatsoever. Now I should certainly like to know how from the conjunction of, say,  $x = y \vee x \neq y$  and  $x = z \vee x \neq z$ , one can derive  $x = z$ .

Unless this can be done, the formal condition  $x = z$  can never be treated as a necessary condition for treating a relation as transitive when it happens to be symmetrical also. Further, such a step will make the transitivity of a relation depend upon the prior ascertainment of its symmetry—a situation which may not be welcome to many in the field of logic. In fact, the usual formal presentation of a transitive relation completely ignores its tacit dependence on the symmetry or asymmetry of the relation concerned. On the Russellian argument, it is the symmetrical character of these relations that forbids their transitivity. But, paradoxically, there is no formal, necessary exclusion between symmetry and transitivity. And, hence, it seems quite strange that a relation should be denied its transitivity just because it happens to be symmetrical also.

Another strange anomaly regarding this argument of Russell may be noticed. Normally, when we speak of transitivity, symmetry, or reflexivity, we speak of them as characteristics of rel-

ations. Given a relation, we can know whether it has these characteristics or not. In Russell's argument, however, it is not the nature of the relation that determines its transitivity, but some such reason as 'x and z may be the same person' or 'x and z may be the same number'. Now, a relation is certainly considered to be transitive when it holds true in all cases and intransitive, when it is false for all the cases and non-transitive, when it holds in some cases and does not hold in others. The relations which Russell is talking about are, then, non-transitive relations. But even among non-transitive relations, they are of a peculiar kind. They are not like, say, 'being friend of' or 'loves' which are non-transitive by their nature. If we cannot say anything about x's being a friend of z or x's loving z when x is the friend of y and y is the friend of z or when x loves y and y loves z, it is because of the nature of 'loving' or 'being friend of' and *not* because of the possible identity of x and z. In Russell's examples, on the other hand, the *only* case where transitivity fails is when there are only two actual terms, one taken twice over for the purpose of counting them as three. This, I suggest, is an entirely different reason from the usual one on grounds of which certain relations are held to be non-transitive.

The denial of transitivity to a relation which fails to be transitive *only* in the case when x and z happen to be the same thing seems a very forced position. It can, of course, be held if one *chooses* to hold it. But there seems little reason to hold such a position. At least, none seems to have been given by Russell or Bhattacharyya or anybody else. In all the instances where z is not the same as x, the relations I referred to in my paper will be considered transitive by everybody including Bhattacharyya and Russell. Bhattacharyya has taken me to task for my concrete example and said: 'Thus it cannot be insisted that "at least in this particular instance, symmetry and transitivity do not lead to reflexivity", for, even in this particular instance, difference has not been shown to be transitive'. He is right in pointing out the unstated assumption, but will he not consider the countless examples where this assumption is satisfied as 'particular instances', where 'symmetry and transitivity do not lead to reflexivity'? In such an example, for instance, as 'Chairs are different from elephants' and 'Elephants are different from lions' the relation is obviously tran-



sitive and leads to the conclusion 'Chairs are different from lions'. Bhattacharyya can, of course, object once again that we are *assuming* that 'chairs' and 'lions' are names of different objects. But he has forgotten that in all such examples we are talking of *concrete* examples where 'chairs' and 'lions' are not variables but designate what they are commonly supposed to designate. The proper question is, 'Are these relations transitive in all those cases where  $x$  and  $z$  are not the same entity?' The answer, I think, can be little in doubt.

Bhattacharyya, I am afraid, has not been quite fair to me in some parts of his reply. For example, when he quotes me as saying, 'In the usual Russellian argument, it is the symmetrical character of the relation that permits the identification of  $x$  and  $z$ ', he has forgotten to add 'though he has nowhere explicitly stated it to be so'. Similarly, when he blandly states that my assertion 'the argument for denying their transitivity *has nothing to do* with the symmetrical, asymmetrical or non-symmetrical character of these relations' is wrong. I was obviously referring to the argument 'x and  $z$  may be the same thing' as given by Russell. That 'different variables may have the same values' is too generalized an argument to be specifically restricted to any particular domain. In fact, Bhattacharyya himself, in his whole discussion, discusses the generalized substitution rule and hardly relates it to the symmetrical condition in this case. The generalized substitution rule is not *necessary* for transitivity. Why, then, does it become *necessary* if the relation is symmetrical in character?—this is the question which neither Russell nor Bhattacharyya has discussed. At another place, Bhattacharyya thinks that I am definitely mistaken when I say that the acceptance of my rule 'would hardly disturb anything except the belief that symmetry and transitivity necessarily involve reflexivity'. And in order to make the mistake apparent, he tries to show that the proposition 'John is different from Harry' will follow from any proposition or set of propositions, including those which are false or meaningless in character. But surely it is an elementary point in the logic of implication that a true proposition (not merely those which are analytically true) is implied by every proposition and that a false proposition implies every other proposition.

These are, obviously, minor points and do not matter much to the central issue of the discussion. The central issue, I suggest, is whether a relation can be denied transitivity if it fails to be so *only* in the case when  $x$  and  $z$  refer to the same object.



## THE SYNTHETIC A PRIORI—SOME CONSIDERATIONS

Most thinkers, today, seem agreed about the untenability of the synthetic a priori. That there can be no propositions about matters of fact which possess a logical necessity about them, seems beyond all dispute. Recent studies in the nature of logic and the analysis of empirical propositions seem to have established this beyond all doubt. But those who have no doubt in the matter do not seem to see that something they hold to be equally obviously true is completely incompatible with it. The purpose of this paper is to focus attention upon something which has necessarily to be denied by anyone who wants to deny the synthetic a priori. In other words, I want to point out what I consider to be the essence of the synthetic a priori and suggest that unless one is prepared to deny it, one cannot deny the synthetic a priori.

The issue of the synthetic a priori is concerned primarily with the question whether there are any necessary relations among matters of fact. A denial of the synthetic a priori is, thus, the denial that there are any necessary relations between any facts whatsoever. Now, it is commonly conceded by everybody that propositions designating facts can be *necessarily* related to each other. In fact, the hypothetico-deductive-verificational method of science in its usual formulation presupposes this. A hypothesis, in order to be verified, has certain consequences drawn from itself in terms of certain possible sense-experiences. If the consequences are found to be the same or similar to those that are expected, the hypothesis is said to be confirmed. If they are different, the hypothesis is given up as inadequate or false. The relation between the hypothesis and the consequences is supposed to be necessary, for unless it was conceived to be such no necessity would be felt for revising the hypothesis when the conclusion was false.

The relation between the premises and the conclusion is the prototype of all necessary relation. Unless, therefore, we are prepared to assert that from empirical propositions designating matters of fact no conclusion can be drawn, we would have to admit necessary relations between some sort of facts and certain others. If all facts were contingent not merely in themselves but in their relation to each other, then no argument concerning facts would be possible, for the form of an argument is always of the nature of 'if...then', i.e., if something is accepted, then something else has to be accepted also and if something is false then something else has to be false also. The usual Humean argument that all relations between facts are logically contingent should lead to the conclusion that even if the fact designated by the conclusion were not to be the case, the fact or facts designated by the premiss or premisses could be the case. This, obviously, is hardly accepted by anybody. The whole logic of verification or even confirmation tacitly depends upon the denial of this conclusion which seems necessarily to follow from the Humean argument concerning the logical contingency of all relations between matters of fact.

The difficulty may be sought to be met by pointing out a confusion in what I have been trying to say. It may be suggested that 'relations between propositions' should not be confused with 'relations between facts'. Just as a proposition may have certain properties without the facts designated by the proposition having that property also, so a proposition may be related to other propositions without the facts designated by those propositions having that relation also. The proposition 'Grass is green', for example, can be true or false without the green grass being true or false. Similarly, the two propositions 'Francine is taller than Shail' and 'Shail is taller than Sujata' together necessarily imply the third proposition 'Francine is taller than Sujata' without, in any way, involving a necessary relation between the facts designated by the propositions concerned. It may be contended that the property of 'logical necessity', like the other property of 'truth' belongs *only* to propositions (i.e., the analytic ones) or to relations between propositions and not to facts in themselves or in their relations to one another.

The analogy with the 'truth' predicate, on which this objection



seeks to base itself, will be difficult to sustain if closely examined. The truth of a proposition or a sentence is determined by the state of affairs designated by it. In the well-known formulation of Tarski, the sentence 'Snow is white' is true, if, and only if, snow is white. It would follow on this analogy that the relations between propositions (i.e., the premises and the conclusion) will be logically necessary if, and only if, the state of affairs designated by them actually obtains in fact. Contrary to this, however, no one seems to believe that the *necessary* relation between propositions has got anything to do with facts. To put it in another way, logical necessity is supposed to be a *syntactical* notion by those who take truth to be essentially *semantic* in character. By treating the notion of 'logical necessity' as semantic on the analogy of the notion of 'truth', the objection concedes in substance far more than most philosophers who deny the synthetic a priori will be prepared to concede.

In fact, the generally accepted position among philosophers seems to be that propositions may be logically coherent, i.e., necessarily related to each other without being factually true. 'Being necessarily related' does not entail 'being factually true', though 'being factually true' is supposed to entail at least 'being not logically incoherent'. No one, as far as I know, maintains that two propositions can be factually true and yet be logically incoherent, though almost everyone maintains that two propositions can be logically coherent without being factually true. 'Not being logically incoherent' does not mean in the case of propositions designating matters of fact 'being logically coherent', i.e., being necessarily derivable from each other.<sup>1</sup> The propositions may be factually true without being necessarily derivable from each other or from any other set or sets of propositions.

The analogy of 'logical necessity' as a characteristic of relation between propositions and 'truth' as a property of propositions will, thus, have to give up an essential point with respect to that which makes them true or logically necessary. 'Logical necessity' would be an intrinsic property of the relation between proposi-

1. This, it should be noted, is a strange situation which normally does not obtain in pure logic. It resembles the case argued for by Gödel where a well-formed formula can always arise in a system which can neither be logically coherent nor logically incoherent within the system.

tions while 'truth' would be their extrinsic property in the sense that it will not depend solely on the nature of the propositions themselves. This would, in effect, reduce itself to the usual distinction between analytic and synthetic judgments or between syntactic and semantic properties of propositions with which we started. The point, then, remains as to why the logical relations between propositions are so intimately and inextricably involved with what may be called the 'truth-relations' between propositions. Why are they not *completely* independent? Why, in other words, if a conclusion is false and if the derivation is logically necessary, should the premises be false also? Why can it not be possible, to put our point in the strongest possible manner, for the conclusion to be false and the premises true or *vice versa*? What is the reason that makes everyone think otherwise? And, last of all, what is involved in this belief which almost everyone seems to share with everyone else?

The only way in which such a belief and such a practice can even possibly be justified is by appeal to the implicit assumption that facts have a logical structure which, whatever may be their specific nature, they can never violate. This assumption is so opposed to the whole atmosphere in which contemporary philosophy breathes that it would seem impossible to make anyone accept it. And I am not interested in making anyone accept it. What, to me, is of more interest is to show that such an assumption is foundational to the belief that pervades the thinking of anyone who thinks about matters of fact. I am not saying that no significant thinking about matters of fact is possible without this foundation, but rather that 'significant thinking' will have to be conceived in a very different manner from the way it is usually conceived by most thinkers who think about these matters.

The point that the relation between premises and conclusion is also, in some way, a relation between the facts designated by them may be objected to in another manner. The premises, it may be said, may not designate facts at all. In the traditional syllogism, for example, in what sense can universal propositions be said to designate facts? 'All men are mortal' includes non-existent men who have not yet been born and men who have ceased to exist. If facts are ineradicably individual, what could possibly be *designated* by propositions which are not singu-



lar in their nature? If facts are inevitably those which are present, i.e., actual, what happens to propositions which are about the past or the future? Do class names designate facts? If not, what happens to all the class-propositions which occur in so many arguments about matters of fact? If, on the other hand, we accept that class names do designate facts, what about the  $n$ th order classes whose members are not individuals but classes of the order<sup>n-1</sup>? Thus, even ordinary declarative sentences may not in all cases designate facts as many have usually supposed.

Similarly, compound propositions formed by connectives raise the question whether the connectives are purely formal in character or whether they do designate any *actual* relation between the facts designated by the elementary propositions concerned. In the compound proposition 'If the metal is heated, then it will expand' the connective 'if, then' seems to designate an actual relation between 'the heating of the metal' and 'its expansion'. On the other hand, in such a compound proposition as 'if Socrates is a pig, then I am a fool' no actual relation seems to be designated by the connective 'if, then'. Even if following Charles Morris, the logical operations are interpreted as behavioural it does not follow that they designate actual relations between matters of fact. Rather, they designate what may be called 'behaviour-expectancies' with respect to the facts designated by the constituent elementary propositions.

The point, of course, is not so simple as that. Why should 'behaviour-expectancies' not be regarded as facts? They do designate mental attitudes which can certainly be verified by the usual technique of observing behaviour over a period of time. The connectives, in such cases, may be taken to designate psychological rather than physical facts. But psychological facts are facts all right. In another interpretation, they may be regarded as rules that prescribe what is to be done with respect to the propositions concerned and not as designating any state of affairs, whether physical or psychological in nature.

The question whether universal propositions, class-propositions and compound propositions may be said to designate facts is complicated and depends for its answer on the prior discussion 'what is a fact?' I feel, however, that a settlement of this issue is neither necessary nor relevant to the point I wish to make. If

once it is conceded that the conclusion cannot be false without the premises being false and that the notions of truth and falsity are essentially semantic in character and that the relation between the premises and the conclusion is one of logical necessity, it seems clear to me that the necessary relation between the falsity of the conclusion and the falsity of the premises must be both necessary and semantic in character. To be both necessary and semantic is the essence of the synthetic a priori and, thus, anyone who concedes the three points made above must concede the synthetic a priori also.

The synthetic a priori, I suggest, can only be denied by those who are prepared to assert that a conclusion can be false and the premises true: that there is no *necessary* derivation of one empirical proposition from another; that just as logic has got nothing to do with facts, facts have got nothing to do with logic. These assertions are far more radical than most deniers of the synthetic a priori will be prepared to make. But unless these assertions are made, the synthetic a priori, I think, cannot be denied.

It may be argued that the assertions I consider necessary for the denial of the synthetic a priori are impossible in their nature and that, if conceded, they will make all scientific investigation untenable. I do not think so. In fact, I have tried to argue otherwise.<sup>2</sup> But here I am only interested in showing what is involved in the denial of the synthetic a priori. Those who consider the conditions for its denial impossible must accept it as an inevitable presupposition of human understanding and thought as Kant did. Or they must do far more radical rethinking about the exact nature of understanding and argument about empirical matters of fact than they seem to have done until now. In either case, what is needed is an awareness of the issue and that is all I am interested in focusing attention upon in this paper.

2. See *The Nature of Philosophy*. Chapters III and IV; 'Law of Contradiction and Empirical Reality', *Mind*, April 1957; 'Types of Coherence', *The Philosophical Quarterly*, Scotland, July 1960.



## TYPES OF COHERENCE

The notion of 'coherence' has played a significant role in epistemological discussions since ancient times. However, though subjected to differing interpretations by different thinkers, the deep typical differences immanent in the notion itself seem seldom to have been noticed. Rather, as is usual with most philosophical controversies, each type is noticed by some thinker or other and declared the sole 'real' coherence which alone is significant and meaningful to the thinker concerned. The other types, even if admitted, are declared as abstract or 'unreal' and, thus, the very question of their interrelationships is avoided from the start. It will therefore be our purpose in this paper to draw attention to types of coherences met with and sought for in different fields, as well as a preliminary consideration of the relations holding, or supposed to be holding, between them.

The concept of coherence is primarily derived from the fields of logic and mathematics and continues, in fact, to derive its content and strength from them. The coherence in these realms consists mainly in the observance of syntactical rules in the derivation of conclusions from primitive assumptions in a particular system. The focal expression finds itself in the law of contradiction which states that  $p$  and  $\sim p$  cannot both be true within a system. The consistency or coherence within a logical or mathematical system, therefore, means that  $p$  and  $\sim p$  must not be derivable from the basic assumptions in accordance with the observance of the syntactical rules. In case both  $p$  and  $\sim p$  can be derived within a system, it is generally concluded that the set of primitive assumptions from which we started is incoherent within itself. But it is equally possible that the incoherence may be within the syntactical rules rather than in the set of primitive assumptions from which we start. It can, of course, lie in both: the only possibility which is excluded being that it lies in neither.

It is well known that the non-occurrence of  $p$  and  $\sim p$  within

any set or sets of derivatives is no proof of the coherence of a particular set of primitive assumptions. Unless a generalized consistency-proof is given of a particular set of primitive assumptions nothing can be said about the coherence or incoherence of the set. Similarly, unless a generalized consistency-proof is given of the syntactical rules nothing can be said about their being internally coherent or incoherent. The empirical fact of non-derivation of  $p$  and  $\sim p$  in a number of cases can be no *ground* for the coherence of either the primitive set of assumptions or of the syntactical rules or both.

The coherence of a set of syntactical rules, it should be remembered, cannot be established by sheer inspection, intuition or self-evidence. The obvious reason for this is that coherence is *defined* in terms of non-derivability of  $p$  and  $\sim p$  and unless the impossibility of such a derivation is shown we cannot significantly speak of coherence of the syntactical rules or, for that matter, of the primitive assumptions.

However, the non-establishment of the coherence of a particular set of assumptions or a particular set of syntactical rules does not usually stand in the way of establishing the coherence of a specific derivation within the system. If each transformation is in accordance with the rules, the derived conclusion is said to be consistent with the primitive assumptions from which we started. It is, on the other hand, a relevant question whether a valid specific derivation may be made from a set of assumptions or syntactical rules which is internally incoherent. Usually, it is supposed that such a derivation can be made. At least in the case of mathematics, we start with specific derivations and only later on work towards a completely formalized deductive structure in which everything is strictly derived from a set of assumptions in accordance with certain syntactical rules. Even if such a formalization is achieved, a rigorous consistency-proof may not be available for the axiomatic set or the syntactical rules. In fact, even when an inconsistency is decisively shown by the appearance of  $p$  and  $\sim p$  in some particular derivation, a change in the axiomatic-set is sought in such a way as to preserve the other derivations within the system.

From the logical point of view, however, *all* the derivations within a system which at any point gives rise to  $p$  and  $\sim p$  must



be invalid. The only alternative to this seems to hold that either the axiomatic-set or the syntactical rules or both are not the minimum required for the derivation within the system. For, if this is not the case, *all* the assumptions in the axiomatic-set and *all* the rules in the syntactic-set are bound to be used in every derivation and, thus, make it invalid. Hence, even though the attempt be made to preserve the valid derivations within the new system, they can hardly be considered as strictly *formally* valid within the old system. It is only within the new system that they can even legitimately claim formal validity for themselves.<sup>1</sup>

The intuitive apprehension that each step in the derivation is *in accordance with* the transformation-rules permitted by the system may lead some to contend that 'coherence' ultimately is a matter of intuition even in a deductive system. The apprehension of an instance as falling under a general rule can, however, hardly be regarded as the apprehension of 'coherence'. A specific individual can scarcely be said to 'cohere' with the class to which it belongs. The issue is, to a certain extent, a linguistic one. But it should be remembered that if the meaning of 'coherence' is stretched to cover such a case, then all cases of similarity or dissimilarity would become cases of 'coherence' or 'incoherence'. As, except for sheer identity, every thing is both similar and dissimilar, it will simultaneously be coherent and incoherent in its different aspects.

1. It may be interesting here to note the similarity between our attempt to preserve the specific derivations within the new system and the attempt to preserve the particular empirical truths within a new theory necessitated by the inability of the older one to account for some newly discovered discrepant facts. We are generally more sure of the specific particulars than the generalized theories and, thus, want to retain them in any modification of the theories forced on us by incompatible facts or contradictory derivations. How far the analogy with inductive generalization can be sustained in mathematics is a question that can hardly be discussed here.

There is, of course, a sense in which all the derivations within a system whose axiomatic-set is self-contradictory are valid. A contradictory set *implies* everything and thus also the derivations which are in question. But obviously there is no sense in calling these "*derivations*" since they are not *specifically* derived at all. Their contradictories can equally well be said to be derived within the system. To put the point differently, the falsity of *p* implies the validity of '*p* implies *q*', but *not the derivability* of *q*.

The concept of coherence, as thus formulated, gives little difficulty in the deductive sciences except those concerned with the consistency-proof of the primitive assumptions and the syntactical rules of the system.<sup>2</sup> Once, however, an empirical interpretation is given to the variables, the difficulties start piling up. What exactly can be meant by the coherence of propositions whose variables have the values concerning matters of fact? Does coherence mean derivability within a system or does it mean 'being true together'? In case either of the alternatives is held to be true, what exactly is meant by 'derivability' in a system whose basic postulates are themselves empirical in nature or by 'being true together' when 'being true' is itself an empirical state of affairs?

It is usually supposed that the derivation from the postulates is a purely *deductive* affair and that the question of empirical truth or falsity arises only when the issue of the 'verifiability' of the consequences deduced from the postulational theory is raised. Such a view is explicitly contained in the well-known description of scientific method as being hypothetico-deductive-verification in nature. But deduction from hypotheses, which themselves are empirical, can hardly be strictly deductive in character. It is, of course, held otherwise. Since Aristotle, it has been held that two empirical propositions can deductively imply a third which itself must be true, if the former are true. If, for example, 'All men are mortal' and 'Socrates is a man', then Socrates must be mortal.<sup>3</sup>

This seems so obvious and so self-evident that it has been taken as the classic example of the deductive derivation of an empirical conclusion from two empirical premises. But a closer examination would reveal that *so far as* the empirical values are taken seriously the deduction is strictly not formal and the more

2. It is contended by some that a consistency-proof, by its very nature, can never be given. If such be the case, the difficulty would lie at the very root of mathematics and make it uncertain in its very foundations. However, the contention is not accepted by many others and various attempts have been made to give consistency-proofs of some systems.
3. It may be said that from two assertory propositions we are deriving a necessary one—a derivation which is invalid. The correct derivation should be 'Socrates is mortal' and not 'Socrates must be mortal'. But a logical derivation cannot but be necessary.



we treat it as formal the more we ignore the specific empirical character of the values concerned. It would be interesting to ask what are the 'truth-conditions' of the three propositions in question. If the first is taken as an empirical inductive generalization, its falsification conditions are hardly defined within the system. What would it mean for a man to be not mortal, is hardly clear from the statement 'All men are mortal'. If, for example, Socrates is not yet dead, does it mean Socrates is not a man or Socrates is not mortal? It will, of course, be replied that what the proposition means is that if Socrates is a man, he is *bound to die*. But if it means just this, it can *never* be falsified, for however long a man lives he cannot be said to be not mortal. Further, if 'All men are mortal' is regarded as true by definition, we cannot even call Socrates a 'man' till he is dead and by dying shows that he is a man.

It may be objected that the difficulties we are pointing out belong to the *individual propositions* and not to the *formal relations* between them on which the whole deductive inference is based. But the issue is just this: whether the introduction of the empirical values to the variables in a propositional function affects or does not affect the formal relations between them? The usual view is that they are not so affected and this is paralleled by the general impression that the form of a proposition remains unchanged when we give empirical values to its variables. But such a belief, however widespread, seems to be mistaken as the form, though ostensibly remaining the same, undergoes a subtle transformation. 'All men are mortal' is not the same as 'All S is P'. The former is more the statement of a probable belief which we are prepared to revise in the light of new experience than the statement of a categorical relation holding true of all 'S's' referred to in the proposition concerned. Such a situation can be avoided only by treating the seemingly universal empirical proposition as a definitional statement conveying the linguistic decision not to use the term 'man' of anything which is not mortal.

It may be objected that if the 'form' is changed, then the values are not values of the original variables, but of some new ones. A variable, it may be said, is nothing if it is not the carrier of an identical form through variations in its values. But supposing there are *no* values to fill in *exactly* the bill provided by the so-

called variables, would it affect in any way the formal structures and their interrelationships revealed in modern logic? Logic, of course, will then be *useless* in the sense that it will have no application; but that, as everybody knows, is logically irrelevant. Or, take another possibility. In case only certain *specific* types of values, say those of mathematics, satisfy the variables, then will it affect the validity of the logical forms and their interrelationships? Obviously, it will have only a limited application, but that equally obviously, will have little to do with the question of validity. Or, if *all* values are supposed to satisfy the variables, is this 'all' merely an empirical, accidental 'all' liable to be falsified tomorrow, or a logical, necessary 'all' which cannot be falsified whatever the nature of the objects denoted by the values of the variables concerned? The latter is usually the way in which logicians have wished their 'all' to be understood; but, if so, it will turn into something metaphysical, i.e., utterly useless for purposes of empirical investigation and knowledge. If, like God, it has to be inevitably realized by everything everywhere, then it certainly need not be taken into account to understand the specific structure and interrelationships of empirical phenomena. If, on the other hand, the inevitability of realization is supposed to follow from the syntactical conditions necessary for any possible cognitive language, it may be urged that no such syntactical conditions have yet been discovered and that languages have performed their cognitive functions very well without any such discovery or even in violation of some such supposed discoveries. The logicians have always in such cases taken the easy road either of denying the cognitive function to such languages or of calling them confused or irrational in their nature. But this, as everyone should know by now, is merely a linguistic solution to a non-linguistic problem by deciding to use the term 'cognitive' in only those cases which the logician wishes to call cognitive. A deeper objection, however, relates to the phrase 'any possible cognitive language'. Are we talking of actual empirical languages which have been prevalent in the past, which are used in the present, and which will be used in the future? Or, is it a mere linguistic decision not to use the word 'language' of anything which does not observe the so-called necessary syntactical conditions? In case it is the former, we can never be sure of the necessity of the



so-called syntactical conditions. If the latter, it has got nothing to do with the empirical phenomenon we know as 'language'. The issue, as always, is between the empirical and the logical. The two have got nothing to do with each other and not, as most philosophers have contended, that there is a one-sided dependence of the empirical on the logical though not vice versa.

The empirical has got nothing to do with the logical unless, of course, we believe in the synthetic a priori. It is, therefore, strange to find persons who reject the synthetic a priori still clinging to the presupposition of the logical by the empirical. The formal identity between empirical and non-empirical propositions is a deceptive one. The form does not retain its purely logical character once the variables are given empirical values. Rather, it itself becomes empirical in character and the relations between the propositions become themselves empirical and no longer remain logical in character. That some sentence, or rather sentential function, should follow from another may depend exclusively on the formal rules of transformation and derivation, but that some state of affairs be inferable from some other state of affairs depends not so much on the rules as on the relation between the state of affairs themselves. This relation, however, is and cannot but be empirical in the sense that no mere insight or contemplation or intuition on the state or the rules howsoever formulated will give us the state of affairs supposed to be inferable from it.

The rules of syntactical transformation concerning the use of such connectives as 'or', 'and', 'if then' may be supposed to be immune from this empirical infection. But this is not so. The connective 'and', for example, is logically commutative, but in such a proposition as 'Francine plucked the fruit and ate it', it obviously cannot function in such a way. The propositional conjunction 'p and q' can always be turned into 'q and p', but once the propositions become empirical, it may not be so.

It may be suggested that the situation can be met by making suitable changes in the proposition concerned. Instead of 'Francine plucked the fruit and ate it' we should, for example, write 'At time t1 Francine plucked the fruit and at time t2 Francine ate it'. This obviously could be reversed. But how do we know that it could be reversed? Certainly not because of the logically commutative nature of the connective 'and', but be-

cause of the nature of the relation of times t1 and t2 to the events with which they are associated. This relation, however, is known as empirically as the fact that Francine could not have eaten the fruit before she plucked it. The very necessity of the introduction of times t1 and t2 to make the conjunction commutative proves that it is not the conjunctive connective 'and' which is responsible for the validity of the commutation.

The necessity of the introduction of time-coordinates into the conjunction to make it commutative may be said to follow from the fact that the connective 'and' in the compound proposition 'Francine plucked the fruit and ate it' is not merely a conjunctive connective but an *ordering connective* also. It is the ordering relation suggested by 'and' which negates the commutative character associated with it. Logic, however, is concerned only with the minimal force of the connective 'and', and not with any overtones associated with the everyday use of it. Our daily 'ands' may carry more than the logician's minimal 'and', but they don't carry *less*. For example, no everyday 'p and q' would be false, if we gave the 'and' its minimal force and if p is true and q is true.

Two things may be said with respect to this. First, the everyday use of *and* in 'Francine plucked the fruit *and* ate it' does not only *not* carry the commutative force of the logician's minimal 'and' but goes positively *counter* to it. And this not because of anything in the nature of this 'and', but rather because of the specific *empirical* nature of the activities it connects. Secondly, though in this case the truth of the conjunction depends upon the truth of the conjoined—a characteristic that the minimal 'and' of the logician is *always supposed* to contain—such is not always the case. For example, from the truth of 'I shall be reading Plato' and 'I shall not be reading Plato' I cannot infer the truth of the conjoint statement, 'I shall *both* be reading Plato and not be reading Plato'. If 'shall' is taken seriously, both the individual statements are correct, as it is equally possible that I may be reading Plato or not reading Plato at any future time n-units hence. But the conjoint assertion is false, as both the events while separately possible are *conjointly* impossible. As Prior has written, though in a slightly different context, we will have to deny 'that  $Fnp$  and  $Fnq$



together entail  $F_n K p q$ , i.e., denying that  $CKF_n p F_n q F_n K p q$  is a law'.<sup>4</sup>

Similarly, once the propositional variables  $p$  and  $q$  are given empirical values in a conditional connective, the relation between them remains no more of 'implication' but of 'causation' and 'causation', as everybody knows, is an empirical relation, a 'hornet's nest'. The truth-conditions of an implicative and a causal relation may seem the same, viz., the impossibility of the antecedent being true and the consequent false. But even this will be a 'seeming' only. For, the truth of an implicative relation is completely established if either the antecedent is false or the consequent is true, while nothing can be said about the truth of a causal relation from the falsity of the antecedent or the truth of the consequent. The attempt to assimilate the causal relation to the formal relation of equivalence will not help the matter either. For the mere fact that the two sets of events described by the two propositions occur or do not occur together does not ensure that there *is* a causal relation between them. What is further required for even the relative establishment of a causal connection is the discovery of detailed *empirical linkages* between the two events. That the truth values of two propositions is always the same may generate a *suspicion* that there is a causal relation between the facts or events designated by the respective propositions. But it can never be *more* than a suspicion and unless specific empirical linkages are established, it will only remain a coincidence and not causation.

It will not do to object that the connectives we have been discussing are not logical connectives at all—the confusion being merely the result of the deceptive identity of the symbol meaning two entirely different things in different contexts. Our contention is just that the logical connectives cannot remain purely logical if they connect propositions which are empirical in character.

4. 'Opposite Number'. *The Review of Metaphysics*, December 1957, p. 200. The whole article should be consulted. Prior has been led to a rejection of this law in a slightly different context. But I suggest that the reasons for rejection lie rather in the *empirical* nature of 'futurity' than in any supposedly discovered logical necessity. The situation is, of course, more complex as Prior rejects the law for the past also. But in both cases it is the postulation of an imagined state of affairs, i.e., something *empirical*, which leads him to reject the law.

Even if in some cases they behave the same way, it is purely accidental and more because of the nature of the empirical propositions connected than because of the so-called logical nature of the connective concerned.

The concept of coherence that seemed relatively adequate for logic in particular and the deductive sciences in general thus does not appear to be so very adequate when considered in the context of the empirical sciences. Yet, the empirical sciences do seek some sort of coherence between their different findings and accept or reject theories on some such ground in their investigations. How, then, is this 'coherence' different from the one in the deductive sciences? The difference is usually conveyed by the term 'verification'. Two empirical propositions may be considered coherent if both of them have been verified to be true. This, it should be seen, involves merely the *factuality* of the two events denoted by the empirical propositions and *nothing more*. No further relation between the two propositions is asserted by calling them coherent than their just being true together. The term 'together' does not mean that one is true 'because of' the other, or that the truth of the one can be inferred from the other, but only that both of them have been found to be true as a matter of fact and nothing more. In this sense, and in this sense alone, 'Brutus killed Caesar' and 'E = MC<sup>2</sup>' are coherent with each other.

This, then, is the weakest sense of empirical coherence. The stronger sense would mean that both of the empirical propositions are derivable from the same set of general empirical principles. This is the coherence that Science seeks for—a unified general theory from which each and every fact should be derivable in a series of finite steps. The derivation, however, as we have shown earlier, is itself not logical but empirical in character.

According to this stronger sense of empirical coherence, it would be meaningless to talk of the coherence of two facts or even of two propositions. Only in the context of a theoretical system would it be meaningful to talk of coherence or incoherence. What is incoherent is not the facts or the isolated propositions but the alternative *interpretations* put upon the same set of facts. Different, even conflicting, interpretations are permissible with respect to different sets of facts. Though permissible, it is not considered desirable and is almost always treated as pro-



visional in character. The whole set of facts, however different they may seem in their ostensible appearance, are supposed to be intrinsically homogeneous in the sense that they must be derivable from and be intelligible in terms of the same unified interpretative theory.

Behind the stronger sense, however, there always looms large the weaker sense which in some respects appears more fundamental and foundational to it. In the ultimate count, it is theory that has to change and not the facts. The facts, in fact, are never supposed to be incoherent or incompatible with each other. Even if a unified theory does not emerge for all sets of facts, we do not consider them to be incoherent for this reason. That something occurs or exists is itself taken as a sign that it must be compatible with the whole realm of the occurrent or existent since otherwise it would never have existed or occurred.

Such a sense, it should be noted, is absent in the case of logical coherence. There it only means deductive derivability from a set of assumptions in accordance with certain rules. 'Being true together', independent of the deductive derivability from a set of assumptions according to certain rules, has no meaning in the case of logical propositions. In fact, a logical proposition, unless it is functioning as an assumption in some system, has no meaning in itself. It makes sense only in the context of the set of assumptions from which it is derived according to the given set of rules. In contrast to this, an empirical proposition does make sense in itself without reference to other empirical propositions. Its meaning is understood only in terms of the possible experience to which it refers to and not in terms of some assumptions or definitional identities from which it is derivable by a series of tautological transformations.

The substitution of empirical values for the variables raises, as we have seen, a host of new problems especially concerned with the 'coherence' of the new type of propositions generated by the substitution. But in case any of the values takes an axiological form, problems of an entirely different type arise. These problems, as far as they bear on the issue of 'coherence', may be designated as those that concern 'axiological coherence'. A proposition which has an axiological term as one of its members may be valid in spite of its being empirically false. Thus, like

propositions in the field of logic or mathematics, axiological propositions involve no reference to existence at least as far as their validation claims are concerned. What can, then, 'coherence' mean in their case and what relations, if any, does it have to the logical and empirical 'coherences' which we have already discussed?

The coherence between any two propositions which have an axiological term as one of the values of their variables seems to lie neither in their being logically, empirically or even axiologically derivable from some generalised axiological principles or even in just being positively valuational together. Taking the latter point first, it seems obvious that two valuational propositions may each be valid and yet be valuationally incompatible with each other. Being valuationally or axiologically valid does not ensure, as being empirically valid does, that the two are coherent with each other. That 'A is good' and 'B is good' does not ensure that they are not incompatible 'goods'. Or that 'A ought to be pursued' and 'B ought to be pursued' does not ensure that both can be or even *ought* to be pursued together. The point, as we shall see later on, concerns the basic heterogeneity of the realm of values as against the postulated or presumed basic homogeneity of the world of facts.

The realm of values, however, seems even different from that of logic or mathematics where a theoretic heterogeneity is permitted by the postulation of different sets of assumptions or derivation-rules if it is so desired or wished. But once the assumptions are made and the derivation-rules adopted one has only to 'see' if the transformations are in accordance with the rules to find whether the conclusion is valid. No further 'seeing' is required to judge the validity of the conclusion in itself. In fact, the 'conclusion' has no 'in-itself' about it: it can only be understood and judged in terms of the premise of which it is the conclusion. Value judgment, on the other hand, has intrinsically the character of 'in-itself' and refuses to be judged in the light of any other judgments—no matter even if they be valuational themselves.

Such an intrinsic autonomy of the individual judgment is shared by the empirical judgment also. This, in fact, is what is ultimately meant by the 'verification-criterion' which we have



already referred to as the weaker but more fundamental and foundational sense of 'coherence' in the empirical field. But the autonomy in the realm of values seems to be far greater than in the empirical realm. The reason for this perhaps lies in the situation that facts are far more closely related to each other than values. Values seem related only by the relation of 'higher' and 'lower' which itself is valuational in nature. Facts, on the other hand, are related by causality and thus are more intimately dependent on each other. In Logic, the relation is closest and thus removes all autonomy of the individual proposition within the system. The autonomy here is only of the system and not of the individual proposition within it, as is, to some extent, the case with empirical propositions and, to the fullest, with the valuational ones.

The autonomy in the valuational field seems to be far greater than in the empirical one. Of course, the realm of fact seems naturally to divide itself into different parts, each studied by a different science with its appropriate methodology and immanent criteria of validity. But there still remains the hope of a unified general theory from which every fact could be derived and in terms of which each could be intelligibly understood. Even such a hope remains foreign to the field of values. The natural division of values into those of Truth, Beauty and Goodness is far more radical in that it makes the very idea of any unifying or overarching value unmeaning in itself. The division between the physical and the social sciences may perhaps seem as radical in the realm of fact as the tripartite division in the realm of values. But the fact of causal relation between the two realms and the situation that the one is built upon the other mitigate, to some extent, the radical division between them. Such, however, does not seem to be the case with the tripartite division of values. The traditional attempt to find such a unifying value in the concept of God is mistaken since, even if true, it shows only the accidental possession of all the three values by some entity and not any essential relation between them.

The problem of coherence in the realm of values thus becomes doubly difficult. One can meaningfully seek it only in the determinate realms of Truth, Beauty, or Goodness and not in any indeterminate realm of value in general. Even within the deter-

minate realms, the judgment that two different values are coherent will be almost as intuitive a judgment as the judgment that this is a value. The coherence, it should be remembered, does not consist in being derivable, either logically, empirically or axiologically from some generalized axiological principle for the simple reason that value has nothing to do with 'derivation'. The fact that something is 'derived' is less important than the immediate intuitive judgment which declares it to be a value. In Logic, 'derivation' is every thing: in empirical sciences, the 'derivation' is not everything since the derived proposition has its verification independent of that from which it has been derived, though the generalized principle from which it has been derived is not so independent as its adequacy is determined by the verification of that which is derived from it. In values, on the other hand, 'derivation' is nothing, since neither the value of the generalized axiological principle nor the value of the derived axiological proposition have anything to do with each other.

The immediate intuitive judgment which seems to determine 'coherence' so completely in the case of values is not altogether absent from the field of logical or empirical sciences. In logic, it occurs as the immediate judgement determining whether the derivation rules have been observed or not. In the empirical sciences, it takes the form of determining whether the proposition has been adequately verified or not. The element, though present is, however, overlaid with many factors which are in some respects even more important than it. The intuitive judgment, for example, that the derivation rules have been observed does not ensure that the axiomatic set or the derivation rules are themselves coherent in nature. Similarly, the adequate verification of a particular empirical proposition does not tell us much about the generalized empirical principles whose verification we might be seeking.

Something more than mere immediacy of judgment for determining the 'coherence' in the field of values may possibly be desired. It may perhaps be found in the notion of 'being realizable together'. If two values can be realized together, they should be regarded as 'coherent'; if not, they should be regarded as 'incoherent' in their nature.

However tempting such a proposal may seem, it is fraught with



grave difficulties. The 'coherence' or 'incoherence', according to it, will be determined not so much by the nature of values as by the type of empirical world we might find ourselves living in. The issue of 'realizability' is not an issue about values, but about the world of causality in which the 'realization' is to take place. The limitations of this world, it is obvious, ought not to be projected on the realm of values. Further, if the criterion is accepted, the 'coherence' notion would cut across the tripartite division of value and make them coherent in the sense of 'being realizable together'. This, though not a great difficulty in itself, obscures the radical and inescapable division among the realm of values and helps the illusion that ultimately there is some necessary integral unity between them.

A deeper difficulty would lie in interpreting adequately the notion of 'being realizable together'. Many values cannot be realized *together*, not because they are incompatible but because of the limitations of time and attention which are such scarce economic goods that their use in one respect prohibits their use in other directions.

The autonomous multiplicity revealed so sharply in the realm of values is sustained and strengthened if we inquire about its relations to the realms of logic and of fact. Neither logic nor fact seem to have any essential relation to values. To be logical or factual is, obviously, not the same as to be valuational. Except for the values immanent to the domain concerned, viz., the values of Validity and Truth, they have nothing to do with the values of the other two realms, viz., Beauty and Goodness. It may be 'better' that Beauty and Goodness 'exist' but it certainly is not necessary for their being that they would not be values if they did not exist.

There is thus no unitary 'coherence' as many have supposed. Rather there are 'coherences', each of a distinct type belonging to realms autonomous and independent in their nature.

## 8

LYING AND THE COMPLEAT  
ROBOT

Michael Scriven has tried, with great subtlety, to indicate what the complete robot will be.<sup>1</sup> He has even suggested how far one can go in eliciting the answer from the robot himself (or herself)<sup>2</sup> if it is built in a certain way. But I think the question whether a robot can lie requires some further discussion than the extremely casual one given by him in his study.

'Lying' involves telling something different from what one knows to be the case. This, of course, is only a first approximation. Lying is by no means confined to verbal reporting only. One may lie in a hundred ways. The gesture, the expression, the turn of the eye, the inflexion of the voice all may be used for lying even more effectively than the utterance of a sentence. Rather, a sentence which is ostensibly false may be uttered in such a way as to convey just the opposite. Or, equally, a true sentence may be embedded in a context of expressive gesture which turns it into an effective lie. 'Lying', then, is a rather complex affair and needs exploration both on the intontional and the performatory sides before the Compleat Robot may do what it is expected to do.

One need not exactly *know* the case, in order that the lie may be a lie. One's belief for that purpose is sufficient. If I believe that such is the case and try to produce, by whatever means, a belief contrary to that which I believe, this will be regarded as 'lying' even if my own belief is *actually* mistaken. In such an event, it will be difficult to suspect that what I told was a lie, for it will be found to be true and normally taken as having been uttered in that way. It is not, thus, the actual contrariety of what

1. *Dimensions of Mind*: A symposium, ed. by Sidney Hook, New York, 1960.
2. The question of sex in robots has, as far as I know, not been discussed until now.